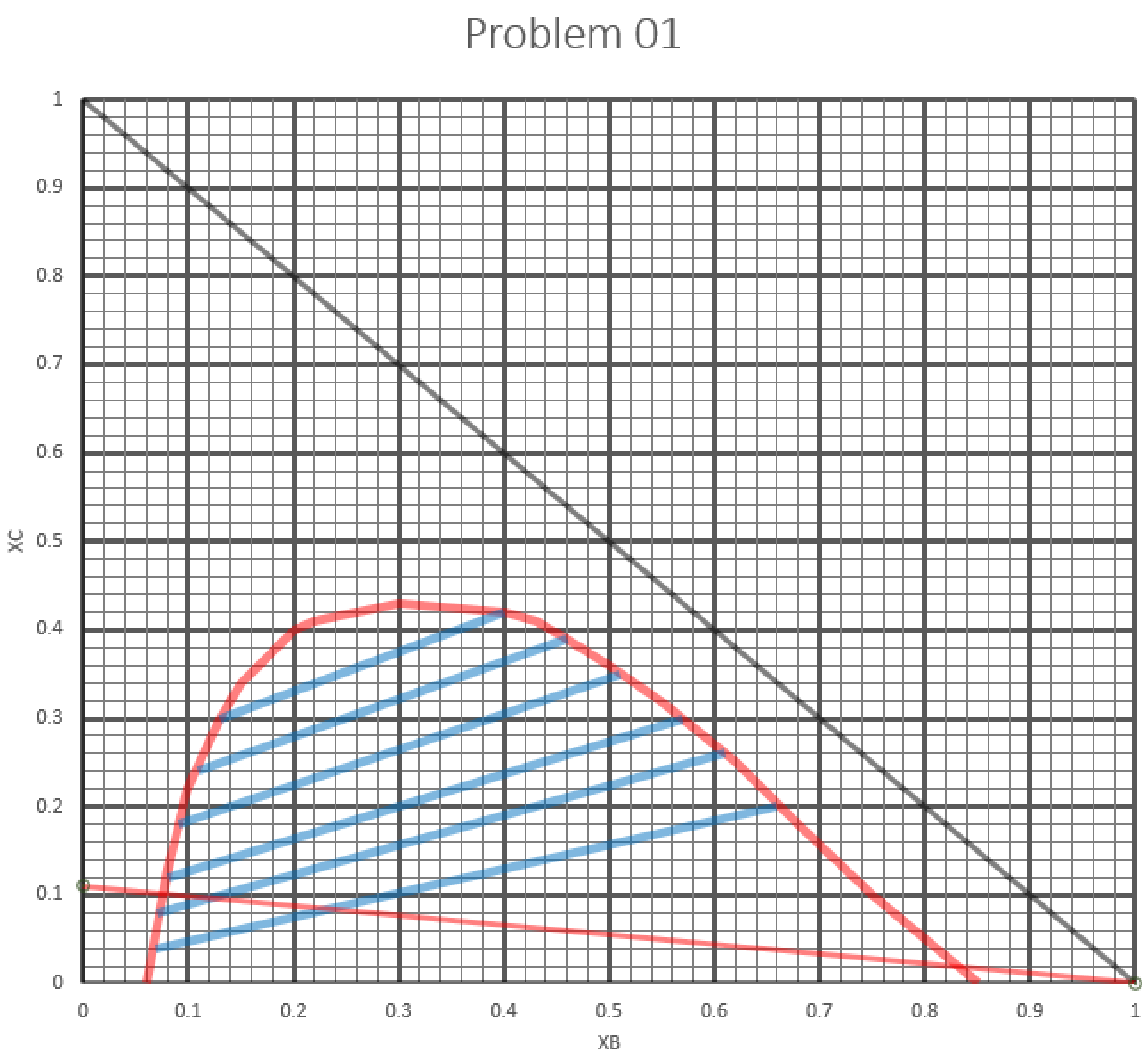
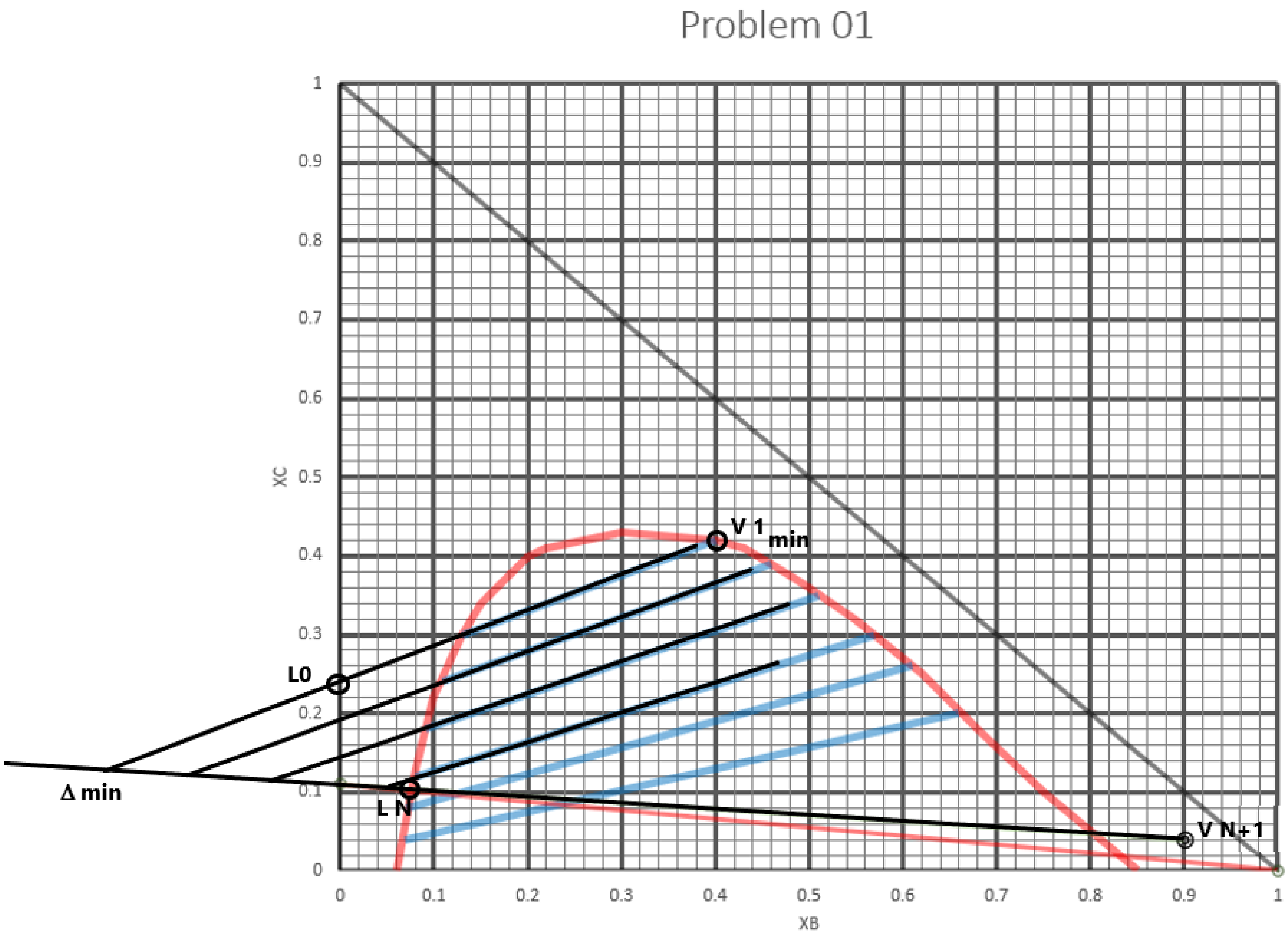
CE 407 Final Exam Solution 12/12/19

1) First we plot **L_N'** and connect it to the pure solvent to locate **L_N** as having $x_C = 0.1$



Next we draw the line from V_{N+1} through L_N and extend it out past the graph. By extending all of the tie lines between L_0 and L_N to this line we can locate Δ_{min}.

Because the topmost tie line extended furthest out, that line is also the line from Δ_{min} to L_0. Where it extends to the other side of the phase boundary is $V_{1\ min.}$


Problem 01
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
XC
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
XB
V 1 min
L0
Δ min
L N
V N+1

Now we will add the lines L_0 to V_{N+1} and L_N to $V_{1\,min}$. The intersection, **M**, is at x_C = 0.20 and $V_{1\,min}$ is at x_C = 0.42

We now want to determine the minimum flow rate of solvent solution:

$$\frac{V_{N+1\,min}}{L_0} = \frac{x_0 - x_M}{x_M - y_{N+1}} = \frac{0.24 - 0.20}{0.20 - 0.04} = 0.25$$

$$V_{N+1\,min} = L_0 * 0.25 = 500 * 0.25 = 125\ \frac{kg}{hr}$$

Because we are using 1.5 times the minimum flow our solvent solution flow will be

$$\boldsymbol{V_{N+1} = 1.5 * 125 = 187.5\ \frac{kg}{hr}}$$

This is the answer for part b.

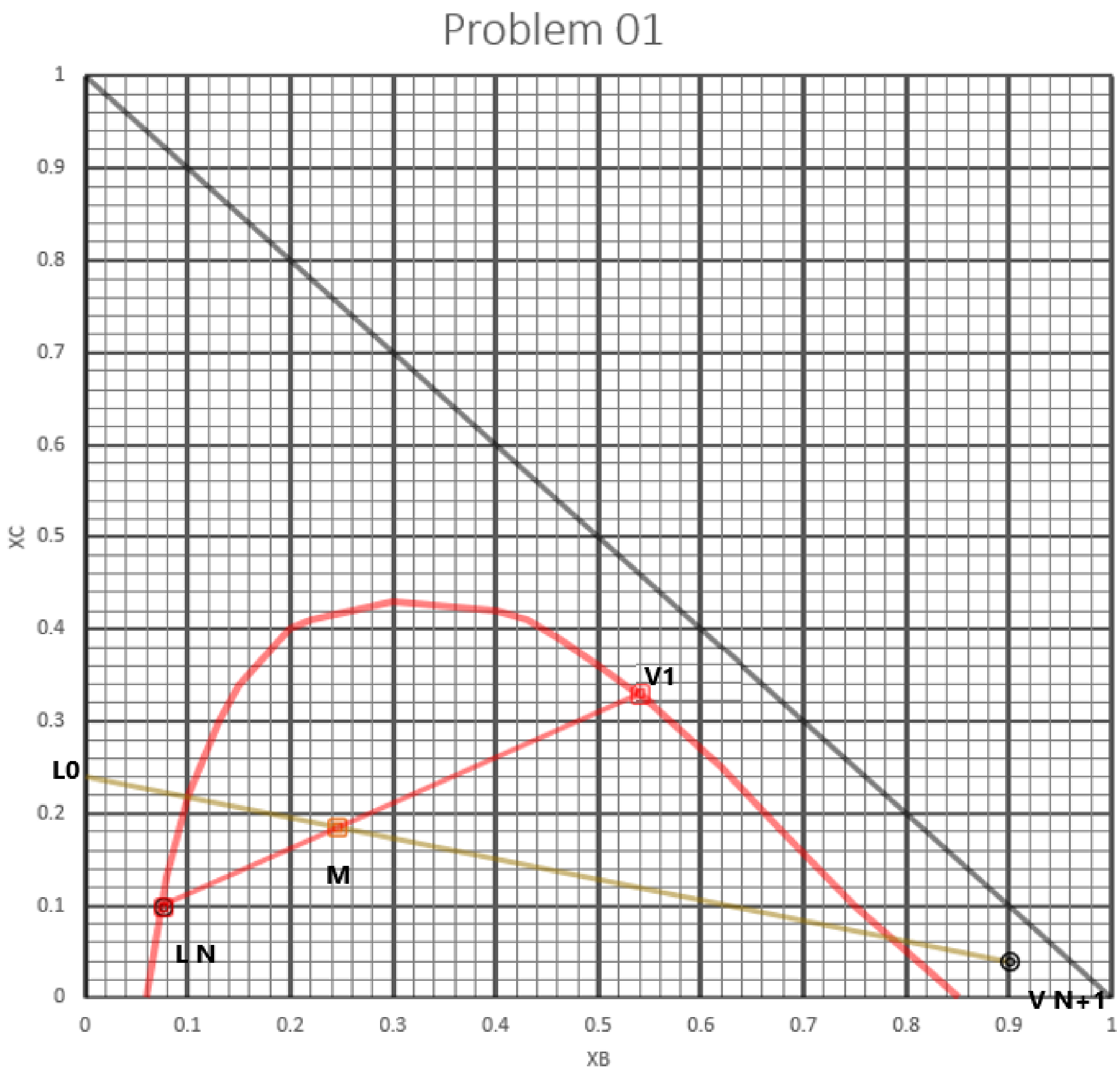
Now we will prepare the next graph to determine points on the operating line:

First determine the new mixing point, **M**:

$$x_M = \frac{F * x_F + S * y_S}{F + S} = \frac{500\,(0.24) + 187.5\,(0.04)}{500 + 187.5} = 0.185$$

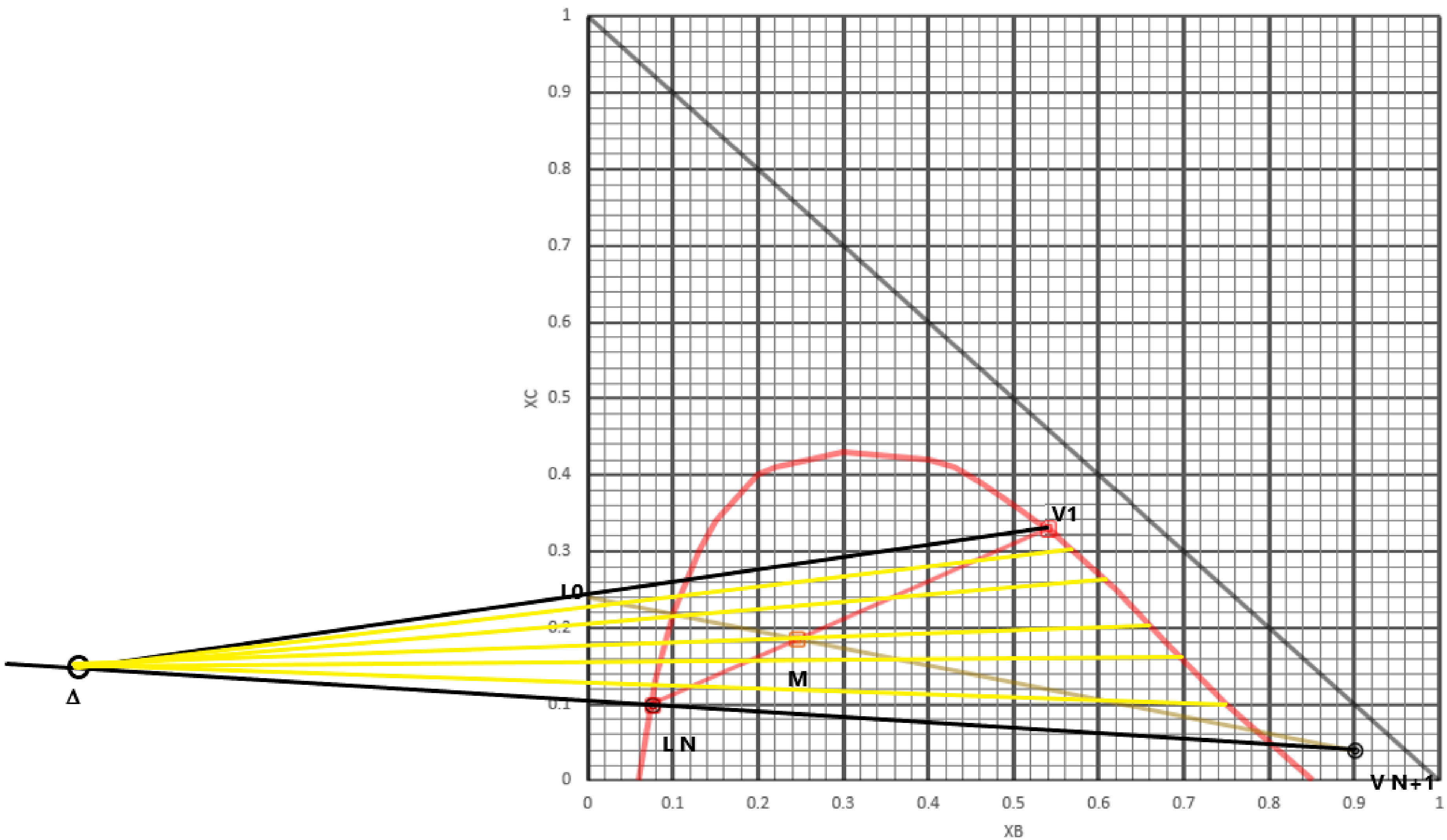
$$x_{M,B} = \frac{F * x_{F,B} + S * y_{S,B}}{F + S} = \frac{500\ (0.0) + 187.5\ (0.90)}{500 + 187.5} = 0.245$$

Plot this point on the line from **L_0** to **V_{N+1}** and extend line from **LN** through **M** until it contacts the solvent rich side of the phase boundary. The point **V_1** is at **x_C** = 0.33.



Now draw lines **L_N** to **V_{N+1}** and **V_1** to **L_0**, extend to outside the graph to locate Δ. Then draw a series of lines to locate points for the operatine lines. (The **x_C** value on the raffinate side of the phase boundary is **x** and the **x_C** value on the extract side of the phase boundary is **y**.)

Problem 01

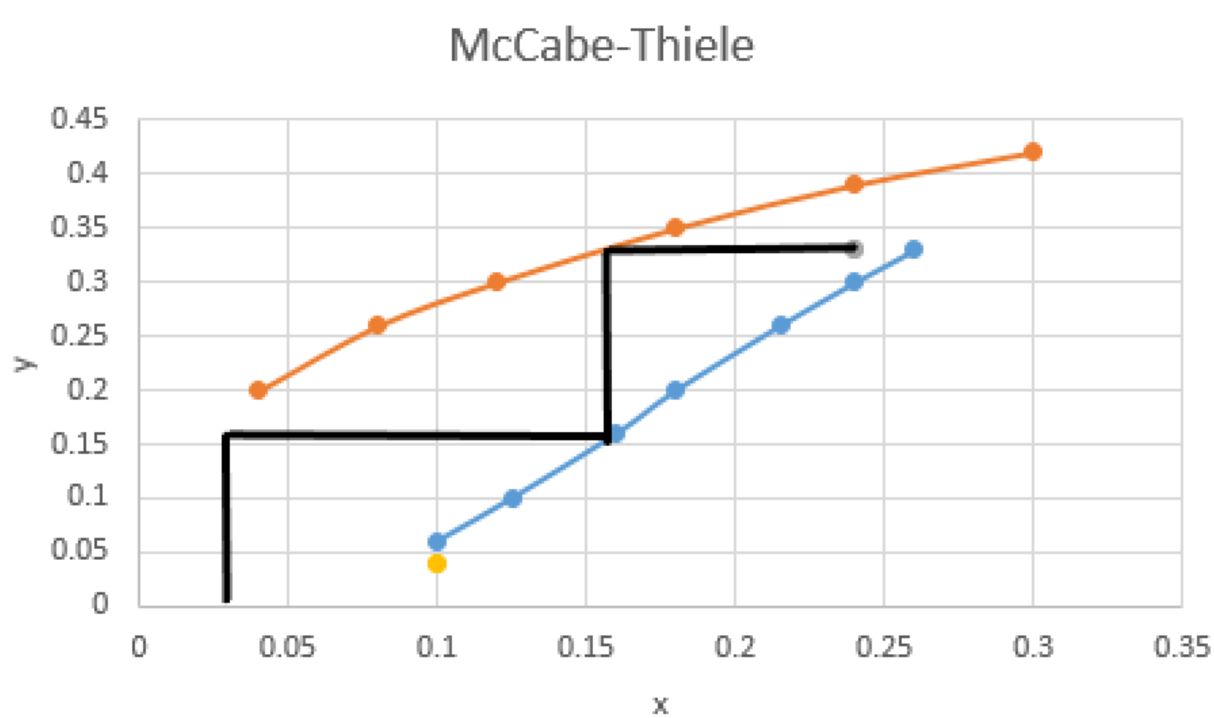


Operating Line Points	
x	y
0.26	0.33
0.24	0.30
0.215	0.26
0.18	0.20
0.16	0.16
0.125	0.10
0.10	0.06

The points for the equilibrium curve by taking $\mathbf{x_c}$ for the raffinate as **x** and $\mathbf{x_c}$ for the extract as **y** (from the equilibirum data provided) for equilibrium curve points.

Equilibrium CurvePoints	
x	y
0.30	0.42
0.24	0.39
0.18	0.35
0.12	0.30
0.08	0.26
0.04	0.29

2 stages are required.



2) The first step is to convert the mass flow given into molar flows:

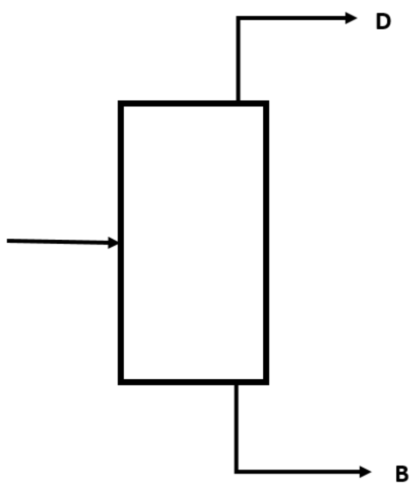
$$1000\ {}^{kg\ total}/_{hr} * 0.45\ {}^{kg\ EB}/_{kg\ total} * \frac{mol\ EB}{106.17\ g\ EB} * \frac{1000\ g}{kg} = 4238.5\ {}^{mol\ EB}/_{hr}$$

$$1000\ {}^{kg\ total}/_{hr} * 0.55\ {}^{kg\ T}/_{kg\ total} * \frac{mol\ T}{92.14\ g\ T} * \frac{1000\ g}{kg} = 5969.2\ {}^{mol\ T}/_{hr}$$

$$x_F = \frac{5969.2\ mol\ T}{5969.2\ mol\ T + 4238.5\ mol\ EB} = 0.58$$

1 hour basis

10207.7 mole total
5969.2 mole T
4238.5 mole EB
x_F = 0.58



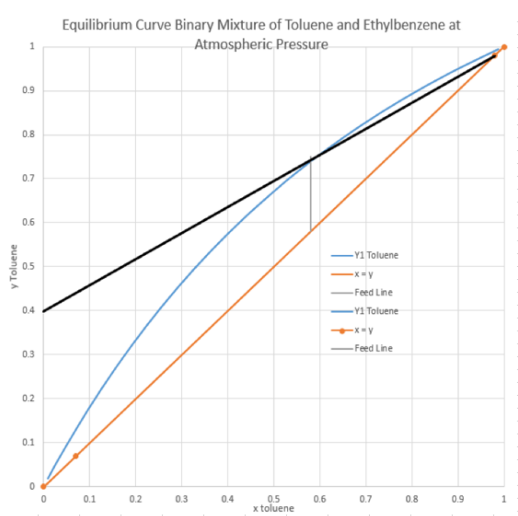
0.95 * 5969.2 = 5670.7 mol T
4238.5 – 4111.3 = 127.2 mol EB
5797.9 mol total

x_D = 5670.7/5797.9 = 0.98

5969.2 – 5670.7 = 298.5 mol T
0.97 * 4238.5 = 4111.3 mol EB
4409.8 mol total

x_B = 298.5/4409.8 = 0.07

The next step is to determine the minimum reflux ratio:



By drawing a line from x_D to the intersection of the feed line and the equilibrium curve, the intercept can be read:

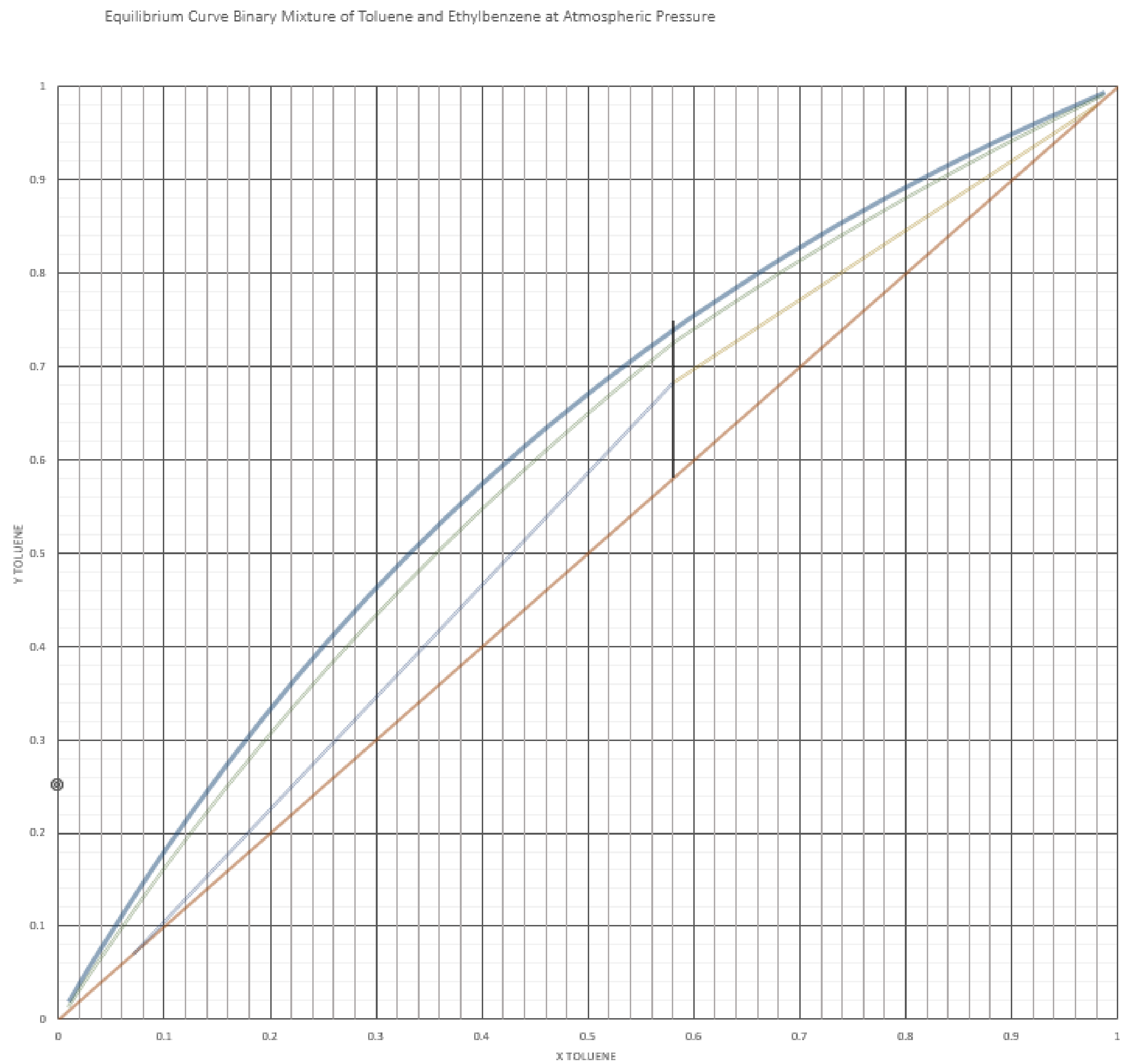
$$\frac{x_D}{R_{min}+1} = 0.4$$

This can be solved for $x_D = 0.98$ to give $\boldsymbol{R_{min} = 1.45}$

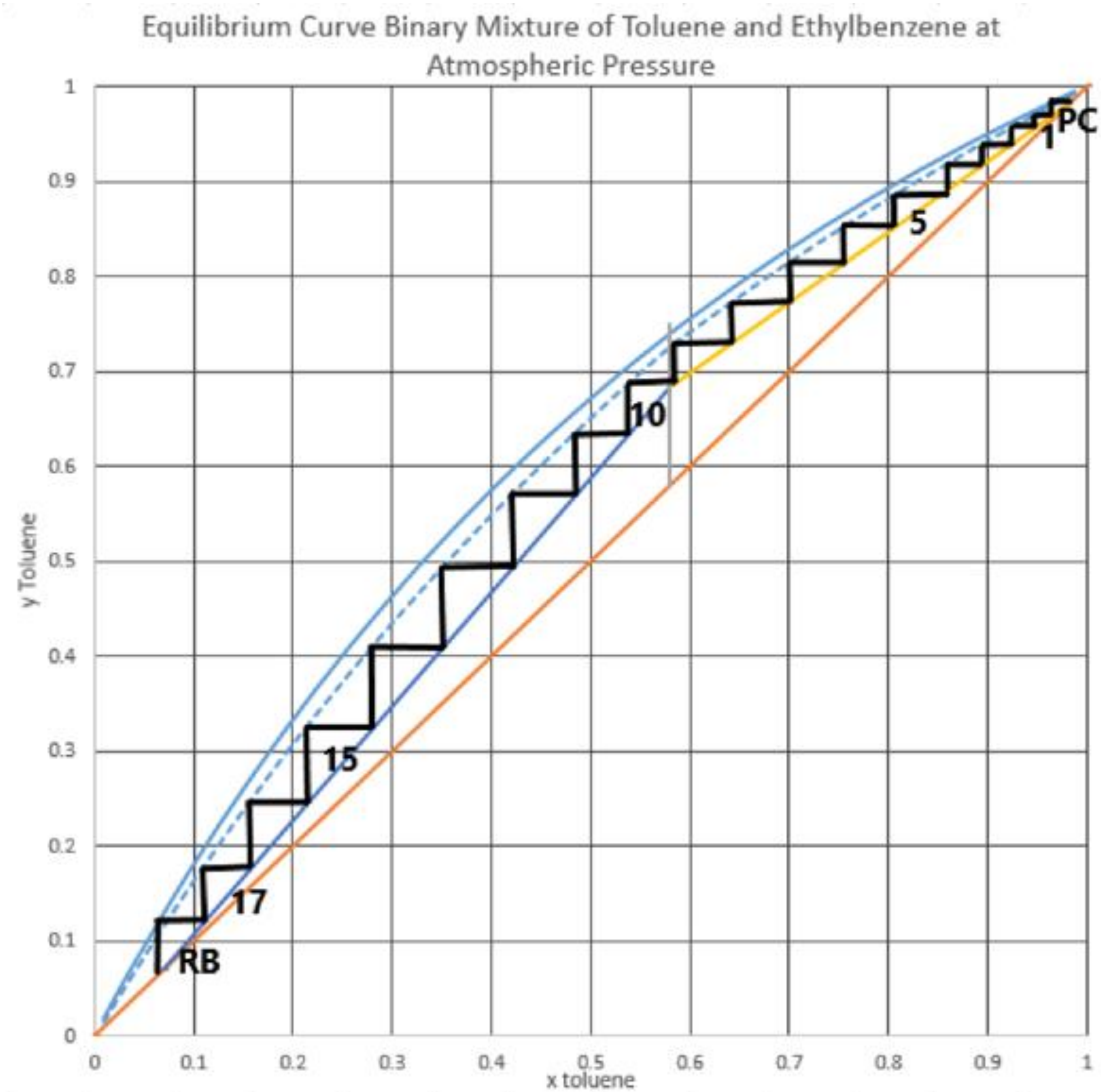
Problem statement says to use $\boldsymbol{R = 2 * R_{min} = 2 * 1.45 = 2.90}$

The new intercept is:

$$\boldsymbol{\frac{x_D}{R+1} = \frac{0.98}{2.90+1} = 0.251}$$



The R operating line can be drawn from the intercept to **(x_D, x_D)** and the S operating line can be drawn from **(x_B, x_B)** to the intersection of the feed line and the R operating line. The effective equilibrium curve is then estimated by marking points 75% of the way from the appropriate operating line and the equilibrium curve.



The McCabe-Thiele steps are drawn. Note that the first step is not a stage but represents the Partial Condenser.

There are 17 stages plus the Reboiler plus the Partial Condenser

The optimal feed location is at the 9th or 10th stage, depending on how one drew everything…

3) Start by converting the mass flow given to amolar flow:

$$\overline{\boldsymbol{MW}} = \boldsymbol{y_{air}} * \boldsymbol{MW_{air}} + \boldsymbol{y_Z} * \boldsymbol{MW_Z} = \mathbf{0.95 * 28.9 + 0.05 * 15 = 28.2}\ \frac{\boldsymbol{lb_m}}{\boldsymbol{lb\ mol}}$$

Moles entering :

$$\frac{5000\ lb_m\ total}{hr} * \frac{lb\ mol}{28.2\ lb_m} = 177.3\ \frac{lb\ mol\ total}{hr}$$

Moles Air:

$$177.3\ \frac{lb\ mol\ total}{hr} * 0.95 = 168.4\ \frac{lb\ mol\ air}{hr}$$

Moles Zapple®:

$$177.3\ \frac{lb\ mol\ total}{hr} * 0.05 = 8.9\ \frac{lb\ mol\ Zapple®}{hr}$$

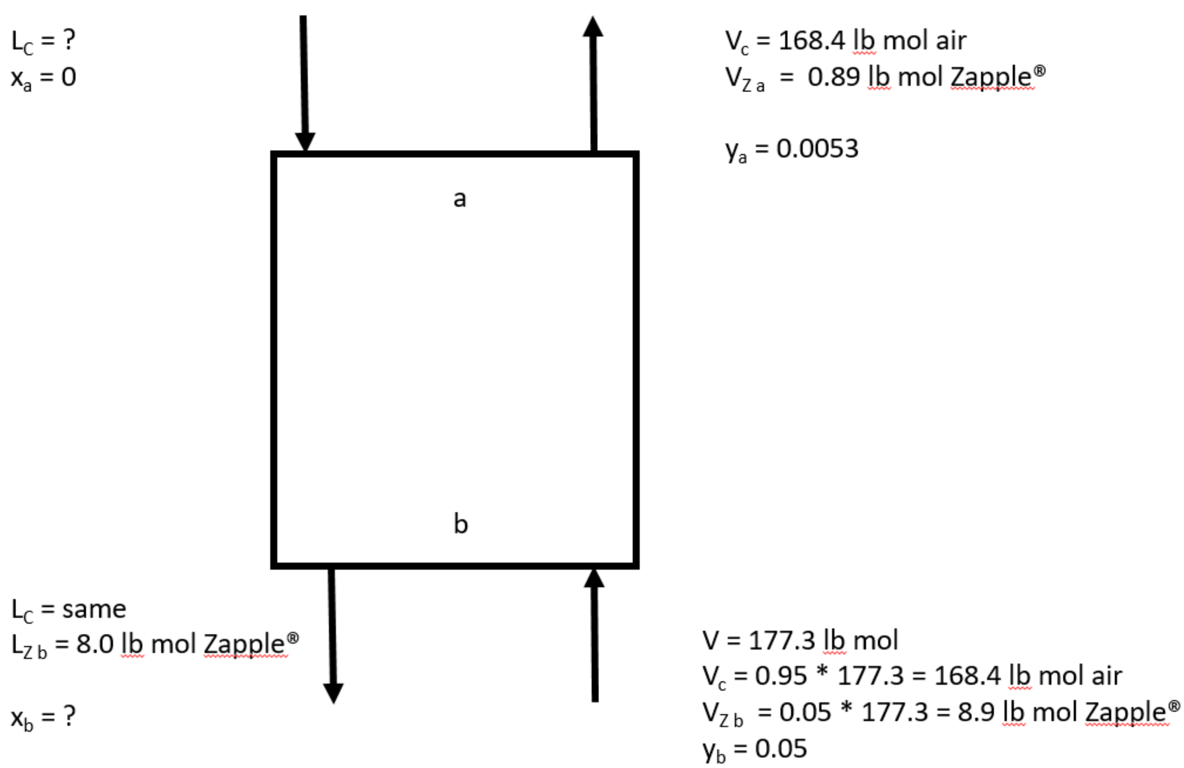
Problem states that 90% of the Zapple® will be removed from the gas stream:

Moles Zapple® in the gas stream at the top, a: = 0.1 * 8.9 = 0.89

Moles Zapple® in the liquid stream at the bottom, b: = 0.9 * 8.9 = 8.0

Mole fraction of Zapple® in the gas stream at the top:

$$y_a = \frac{0.89}{168.4 + 0.89} = 0.0053$$



Because the problem statement indicates that the operating line can be considered linear we can assume that for minimum liquid flow the mole fraction of Zapple® in the exiting liquid is in equilibrium with the entering gas:

From problem statement:

$$y = 0.7\,x$$

$$x_b = \frac{y_b}{0.7} = \frac{0.05}{0.7} = 0.0714$$

The mole fraction in the exiting liquid is:

$$x_b = \frac{L_{Z,b}}{L_{C,min} + L_{Z,b}} = \frac{8.0}{L_C + 8.0} = 0.0714$$

The minimum liquid flow can be solved as:

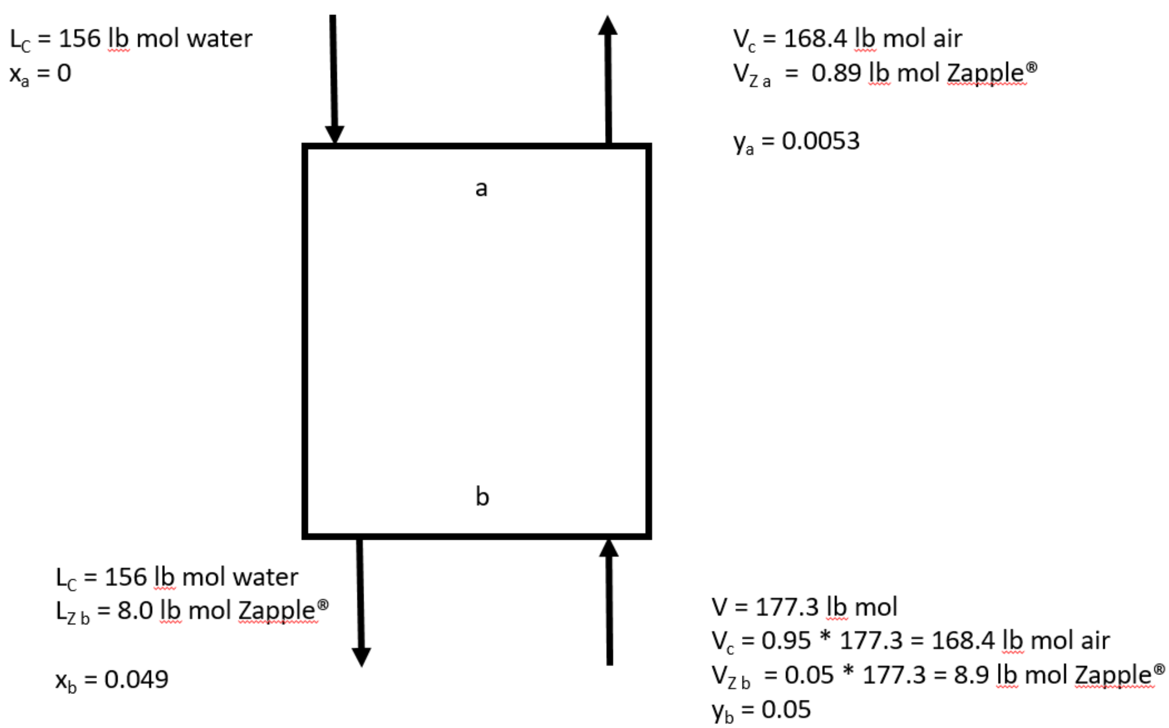
$$L_{C,min} = 104.0\ \frac{lb\ mol\ water}{hr}$$

As per the problem statement, we will be using 1.5 times the minimum flow:

$$L_C = 1.5 * L_{C,min} = 1.5 * 104.0 = 156\ \frac{lb\ mol\ water}{hr}$$

Now we can calculate the mole fraction in the exiting liquid:

$$x_b = \frac{L_{Z,b}}{L_C + L_{Z,b}} = \frac{8.0}{156 + 8.0} = 0.049$$



a. Now we use the flooding correlation given:

$$\Delta P_{flood} = 0.115 * F_P^{0.7}$$

From the data table given we see that for 1 ½ " Plastic Pall Rings **F_p** = 40 and **f_p** = 1.18

$$\Delta P_{flood} = 0.115 * 40^{0.7} = 1.52 \frac{"\ H20}{ft}$$

We are instructed to work at 50% of the flooding pressure drop, therefore we will use 0.75 " water per foot of packing.

To use the attached chart we must calculate $\frac{G_x}{G_y}\sqrt{\frac{\rho_y}{\rho_x}}$

Although we do not know the cross-sectional are required to calculate **G_x** or **G_y**, the ratio of them is the same as the ratio of the mass flows of the liquid and vapor.

Mass flow of vapor:

At bottom = 5000 lb/hr

At top = 168.4 lbmol air/hr * 28.9 lb / lbmol + 0.89 lbmol Z/hr * 15 ln /lbmol = 4880 lb/hr

Average = 4940 lb /hr

Mass flow of liquid:

At top = 156 lbmol water/hr * 18 lb/lbmol = 2808 lb/hr

At bottom = 156 lbmol water/hr * 18 lb/lbmol + 8.0 lbmol/hr * 15 ln/lbmol = 2928 lb/hr

Average = 2868 lb/hr

$$\boldsymbol{\frac{G_x}{G_y} = \frac{2868}{4940} = 0.581}$$

Note: if one had just used the values at the bottom of the tow the ratio would be 0.562, very similar…

The density of the liquid can be approximated as the density of water,

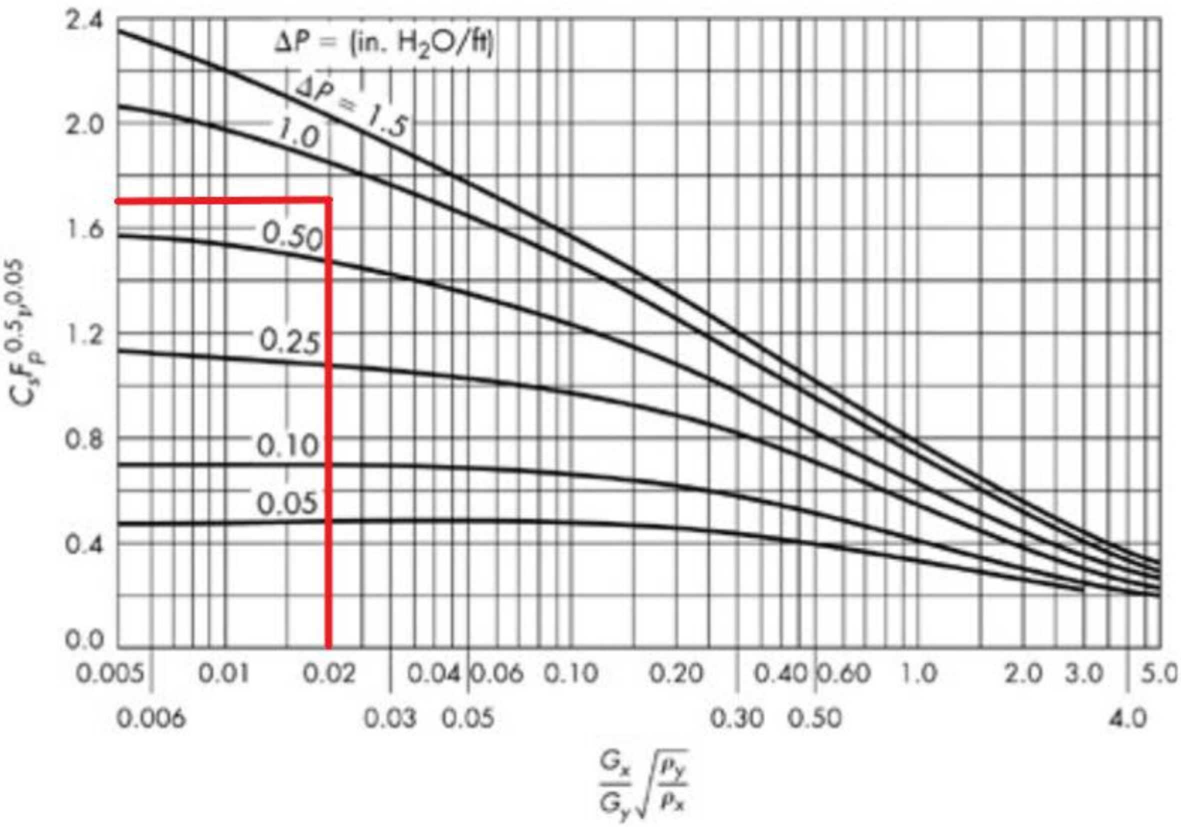
$$\boldsymbol{\rho_x = 62.4\ lb/ft^3}$$

The density of the vapor can obtained using the ideal gas law:

$$\boldsymbol{\rho_y = \frac{\overline{MW}P}{RT} = \frac{28.2\ \frac{lb}{lbmol}\ 1\ atm}{0.73024\ \frac{ft^3 atm}{R\ lbmol}\ 527.67\ R} = 0.07318\ \frac{lb}{ft^3}}$$

Now:

$$\frac{G_x}{G_y}\sqrt{\frac{\rho_y}{\rho_x}} = 0.581\sqrt{\frac{0.07318}{62.4}} = 0.0199$$



From graph we can read (by interpolating between the lines from 0.5 and 1.0 "water/ft) that

$$C_s F_p^{0.5} \nu^{0.05} = 1.7$$

$$C_s 40_p^{0.5} 1^{0.05} = 17$$

$$C_s = 0.269$$

By definition:

$$C_s = u_0 \sqrt{\frac{\rho_y}{\rho_x - \rho_y}}$$

$$0.269 = u_0 \sqrt{\frac{0.07318}{62.4 - 0.07318}} = 0.0323\, u_0$$

$$u_0 = 7.85\, ft/s$$

Calculate largest volumetric flow:

$$5000\ \frac{lb}{hr} * \frac{ft^3}{0.07318\ lb} * \frac{hr}{3600\ s} = 19.0\ \frac{ft^3}{s}$$

The required area is:

$$Area = \frac{volumetric\ flow}{linear\ velocity} = \frac{19.0\ \frac{ft^3}{s}}{7.85\ ft/s} = 2.42\ ft^2 = \frac{\pi\ D^2}{4}$$

$$D = 1.75\ ft$$

b) Now that we have the cross-sectional area we can calculate **G_x** and **G_y** independently:

$$G_x = \frac{2868\ \frac{lb}{hr}}{2.42\ ft^2} = 1185\ \frac{lb}{ft^2\ hr}$$

$$G_y = \frac{4940\ \frac{lb}{hr}}{2.42\ ft^2} = 2041\ \frac{lb}{ft^2\ hr}$$

We can now calculate the height of the transfer unit:

$$H_x = 0.9\ ft \left(\frac{G_x/\mu}{1500\ \frac{lb}{ft^2\ hr}\Big/0.891\ cP} \right)^{0.3} \left(\frac{S_c}{381}\right)^{0.5} \frac{1}{f_p}$$

$$H_x = 0.9\ ft \left(\frac{1185\ \frac{lb}{ft^2\ hr}\Big/0.891\ cP}{1500\ \frac{lb}{ft^2\ hr}\Big/0.891\ cP} \right)^{0.3} \left(\frac{350}{381}\right)^{0.5} \frac{1}{1.18} = 0.68\ ft$$

$$H_y = 1.4\ ft \left(\frac{G_y}{500\ \frac{lb}{ft^2\ hr}} \right)^{0.3} \left(\frac{1500\ \frac{lb}{ft^2\ hr}}{G_x} \right)^{0.4} \left(\frac{S_c}{0.66}\right)^{0.5} \frac{1}{f_p}$$

$$H_y = 1.4\ ft \left(\frac{2041}{500\ \frac{lb}{ft^2\ hr}} \right)^{0.3} \left(\frac{1500\ \frac{lb}{ft^2\ hr}}{1185} \right)^{0.4} \left(\frac{0.75}{0.66}\right)^{0.5} \frac{1}{1.18} = 2.12\ ft$$

Now we can calculate height of overall transfer unit:

$$\boldsymbol{H_{Oy} = H_y + m\,\frac{V}{L} * \,H_x}$$

From problem statement we know that y = 0.7 x and therefore m = 0.7

For V/L we can use:

$$V/_L = \frac{x_b - x_a}{y_b - y_a} = \frac{0.049 - 0}{0.05 - 0.0053} = 1.10$$

Had one used the molar flows at the bottom of the tower V/L = 1.08 would be obtained.

Now:

$$\boldsymbol{H_{Oy} = 2.12\,ft + 0.7 * 1.10 * \,0.68\,ft = 2.64\,ft}$$

Now we will calculate the number of transfer units:

$$\boldsymbol{N_{Oy} = \frac{y_b - y_a}{\overline{\overline{(y - y^*)_{lm}}}}}$$

$y_a = 0.0053$

$y_b = 0.05$

$y_a^* = 0.7 * \,x_a = 0.7 * 0 = 0$

$y_b^* = 0.7 * \,x_b = 0.7 * 0.049 = 0.0343$

$y_b - y_a = 0.05 - 0.0053 = 0.0447$

$y_a - y_a^* = 0.0053 - 0 = 0.0053$

$y_b - y_b^* = 0.05 - 0.0343 = 0.0157$

$$\overline{\boldsymbol{(y - y^*)_{lm}}} = \frac{(y_a - y_a^*) - (y_b - y_b^*)}{\ln\frac{y_a - y_a^*}{y_b - y_b^*}} = \frac{0.0053 - 0.0157}{\ln\frac{0.0053}{0.0157}} = \frac{-0.0104}{-1.0860} = 0.00958$$

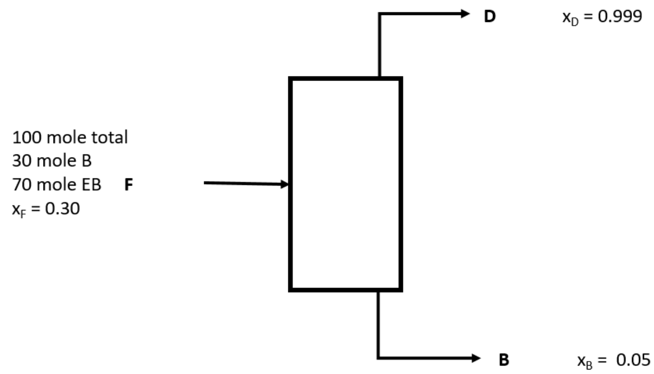
$$\boldsymbol{N_{Oy} = \frac{y_b - y_a}{\overline{\overline{(y - y^*)_{lm}}}} = \frac{0.0447}{0.00958} = 4.67}$$

The required height of the packing can be calculated as:

$$\boldsymbol{Z_t = H_{Oy} * \,N_{Oy} = 2.64\,ft * \,4.67 = 12.3\,ft}$$

4)

1 minute basis



Reflux ratio is given as 1.5

a)

$$\boldsymbol{D = F\left(\frac{x_F - x_B}{x_D - x_B}\right) = 100 * \left(\frac{0.3-0.05}{0.999-0.05}\right) = 26.34\ mol/min}$$

$$\boldsymbol{B = F\left(\frac{x_D - x_F}{x_D - x_B}\right) = 100 * \left(\frac{0.999 - 0.3}{0.999 - 0.05}\right) = 73.66\ mol/min}$$

b) and c)

The R Operating line has the following equation:

$$y_{n+1} = \frac{R}{R+1}\,x_n + \frac{x_D}{R+1}$$

$$y_{n+1} = \frac{1.5}{1.5+1}\,x_n + \frac{0.999}{1.5+1}$$

$$\boldsymbol{y_{n+1} = 0.6000000\,x_n + 0.3996000}$$

Use the Kremser Equation to determine the number of stages required to go from x = 0.9 to x = 0.999

We need to determine the equation for the line which approximates the equilibrium curve in this region:

Line passes through the points (1,1) and (0.9000000, 0.9815065)

The slope will be (rise over run):

$$m = \frac{1 - 0.9815065}{1 - 0.9000000} = 0.1849350$$

The intercept can be evaluated for the point (1,1)

$$y = 0.1849350\,x + b$$
$$1 = 0.1849350 * (1) + b$$
$$b = 0.8150650$$

$$\boldsymbol{y = 0.1849350\, x + 0.8150650}$$ **Equilibrium Line**

Data needed for Kremser Equation:

a evaluated at x = x_D

b evaluated at x = 0.9

At x = x_D = 0.999 the value on the operating line is equal to x_D= 0.999

$$y_a = \mathbf{0.6000000 * 0.999 + \ 0.399600 = 0.9990000}$$

At x = 0.90 the value on the operating line is

$$y_b = \mathbf{0.6000000 * 0.9 + \ 0.399600 = 0.9396000}$$

At x = x_D = 0.999 the value on the equilibrium line is

$$\boldsymbol{y_a^* = 0.1849350 * 0.999 + 0.8150650 = 0.9998151}$$

At x = 0.90 the value on the operating line is (this value is also known from the problem statement)

$$\boldsymbol{y_b^* = 0.1849350 * 0.9 + 0.8150650 = 0.9815065}$$

The number of stages required to get from x = 0.9 to x = 0.999 is:

$$\boldsymbol{N = \frac{ln[(y_b - y_b^*)/(y_a - y_a^*)]}{ln[(y_b - y_a)/(y_b^* - y_a^*)]}}$$

$\boldsymbol{y_b - y_b^* = -0.0419065}$

$\boldsymbol{y_a - y_a^* = -0.0008151}$

$\boldsymbol{y_b - y_a = -0.0594000}$

$\boldsymbol{y_b^* - y_a^* = -0.0183086}$

$$\boldsymbol{(y_b - y_b^*)/(y_a - y_a^*) = {}^{-0.0419065}/_{-0.0008151} = 51.41271}$$

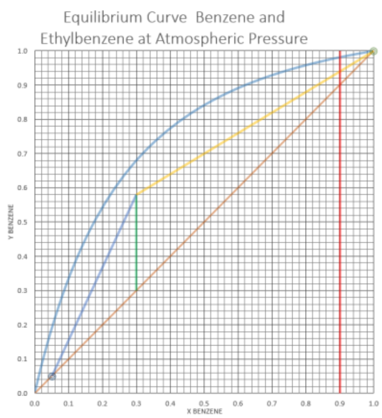
$$\boldsymbol{(y_b - y_a)/(y_b^* - y_a^*) = {}^{-0.0594000}/_{-0.0183086} = 3.244377}$$

$$N = \frac{ln[(y_b - y_b^*)/(y_a - y_a^*)]}{ln[(y_b - y_a)/(y_b^* - y_a^*)]} = \frac{ln[51.41271]}{ln[3.244377]} = \frac{3.939885}{1.176923} = 3.35\ stages$$

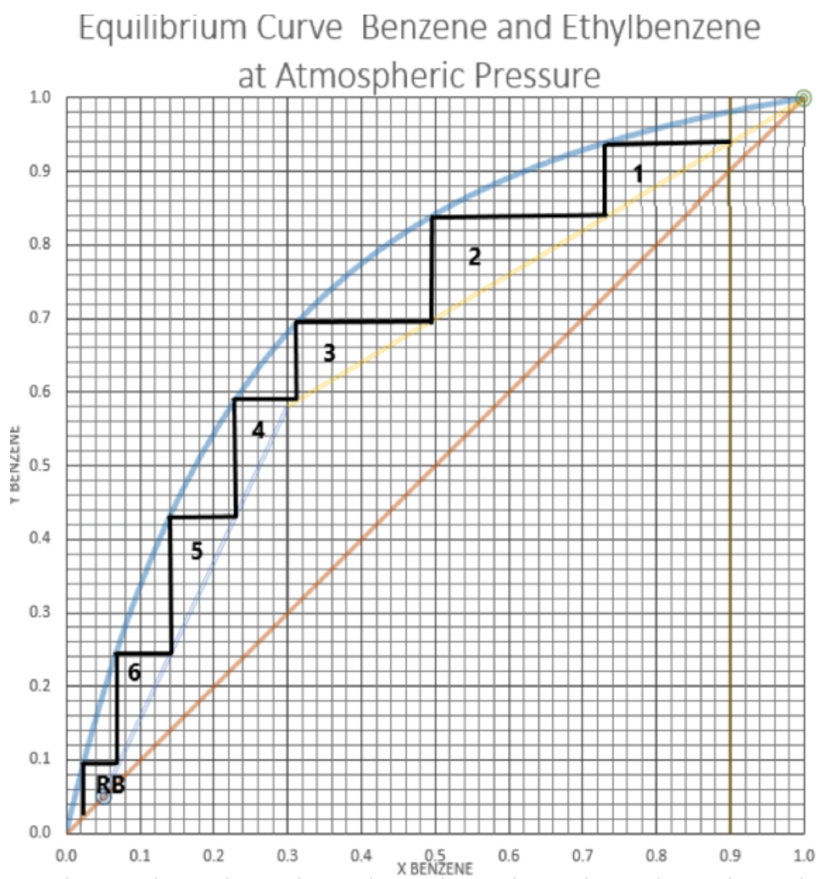
Round up to 4 stages

Now on McCabe-Thiele diagram draw R operating line from (**x_D, x_D**) to the intercept calculated earlier, 0.4 (we don't need all of the significant digits used earlier.

Then draw the S operating line from (**x_B, x_B**) to the intersection of the equilibrium curve and the feed line. Because the feed entered as a saturated liquid the feed line is a vertical line at x = 0.



The steps can then be drawn in:



The required number of steps is 4 + 6 + Reboiler.

Therefore 10 stages plus reboiler.

The feed enters on stage 4 + 4 = Stage 8