

## CE 407 Final Exam Solution 12/12/19

- 1) First we plot  $L_N'$  and connect it to the pure solvent to locate  $L_N$  as having  $x_C = 0.1$

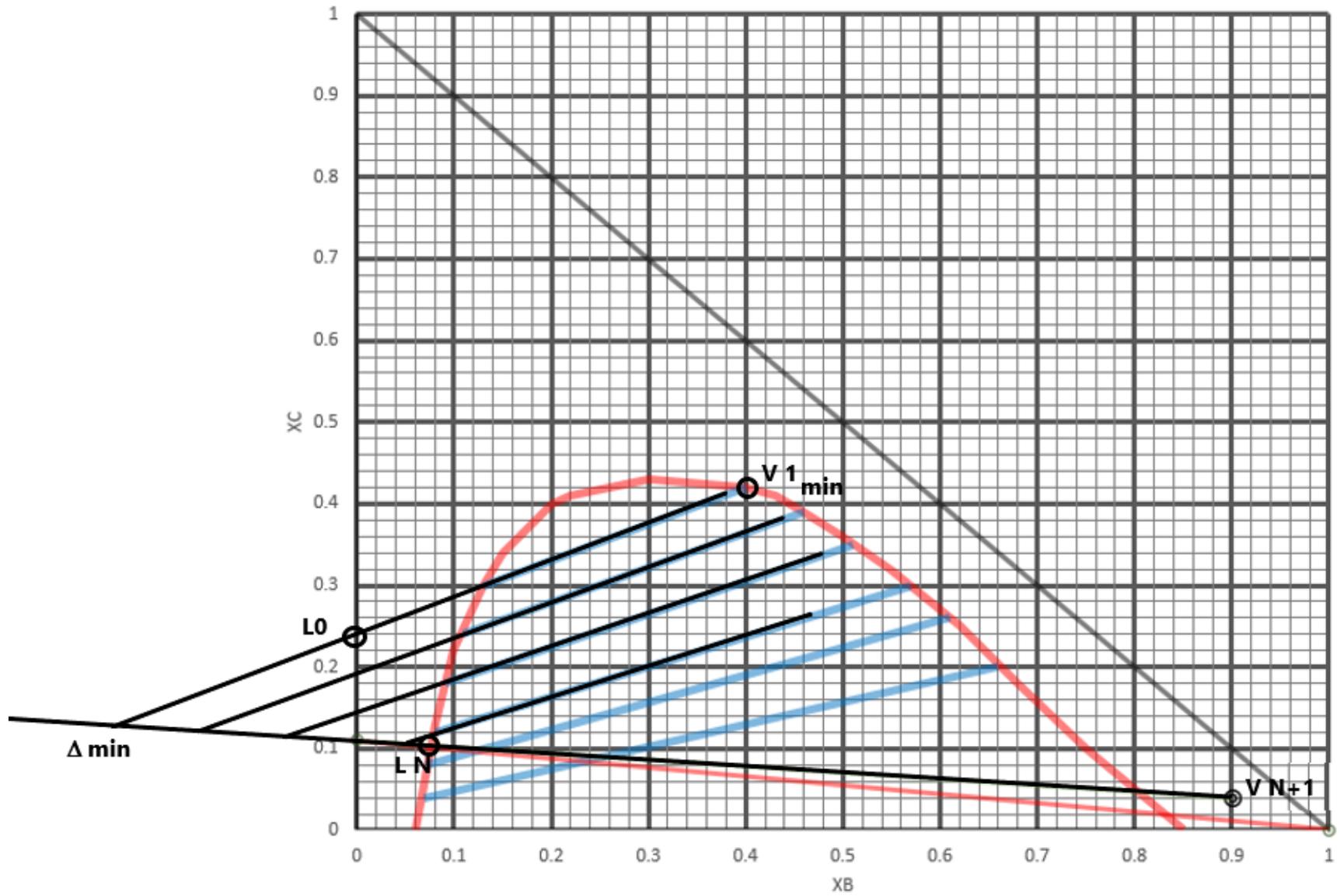
### Problem 01



Next we draw the line from  $V_{N+1}$  through  $L_N$  and extend it out past the graph. By extending all of the tie lines between  $L_0$  and  $L_N$  to this line we can locate  $\Delta_{min}$ .

Because the topmost tie line extended furthest out, that line is also the line from  $\Delta_{min}$  to  $L_0$ . Where it extends to the other side of the phase boundary is  $V_{1 min}$ .

# Problem 01

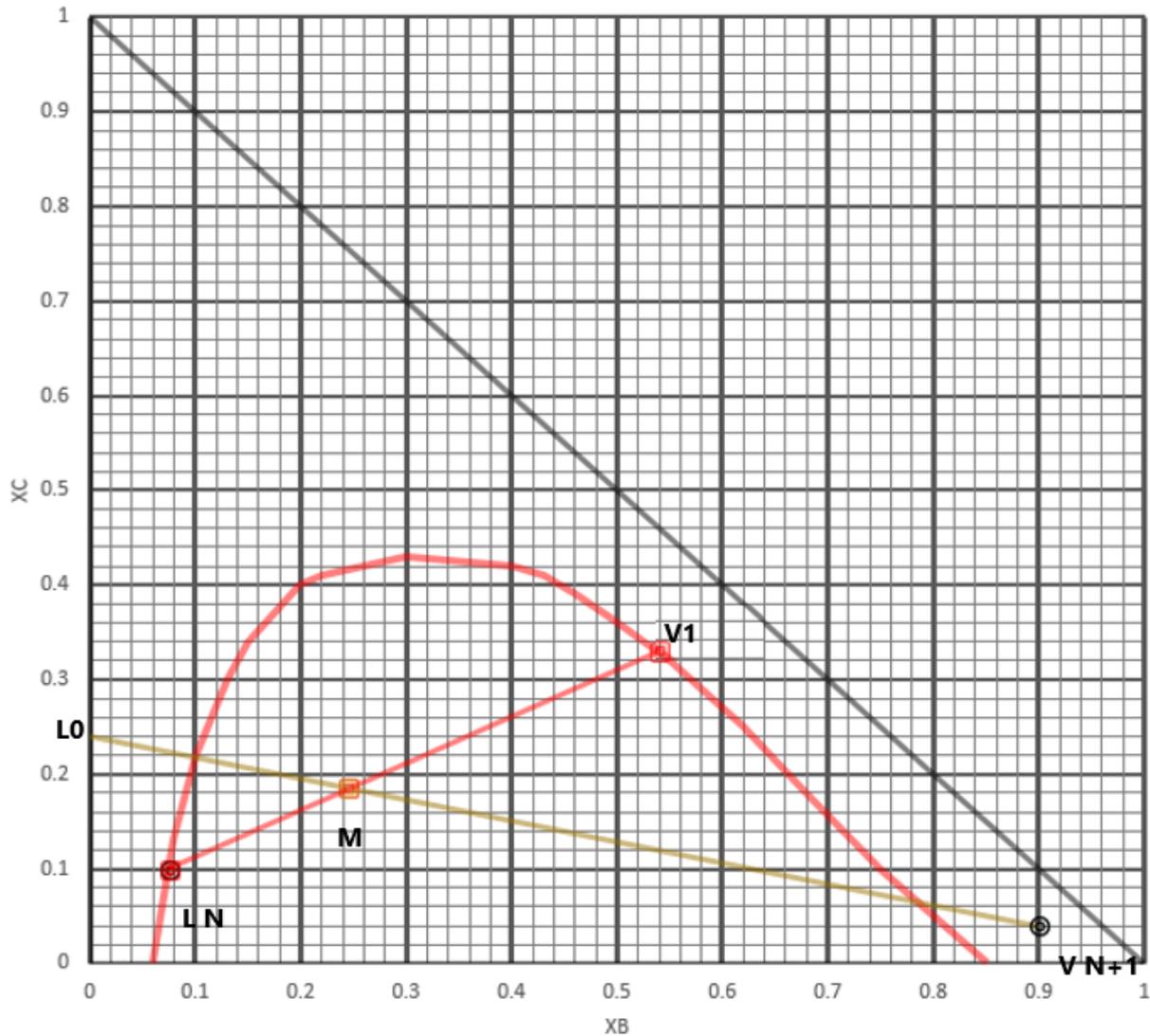




$$x_{M,B} = \frac{F * x_{F,B} + S * y_{S,B}}{F + S} = \frac{500 (0.0) + 187.5 (0.90)}{500 + 187.5} = 0.245$$

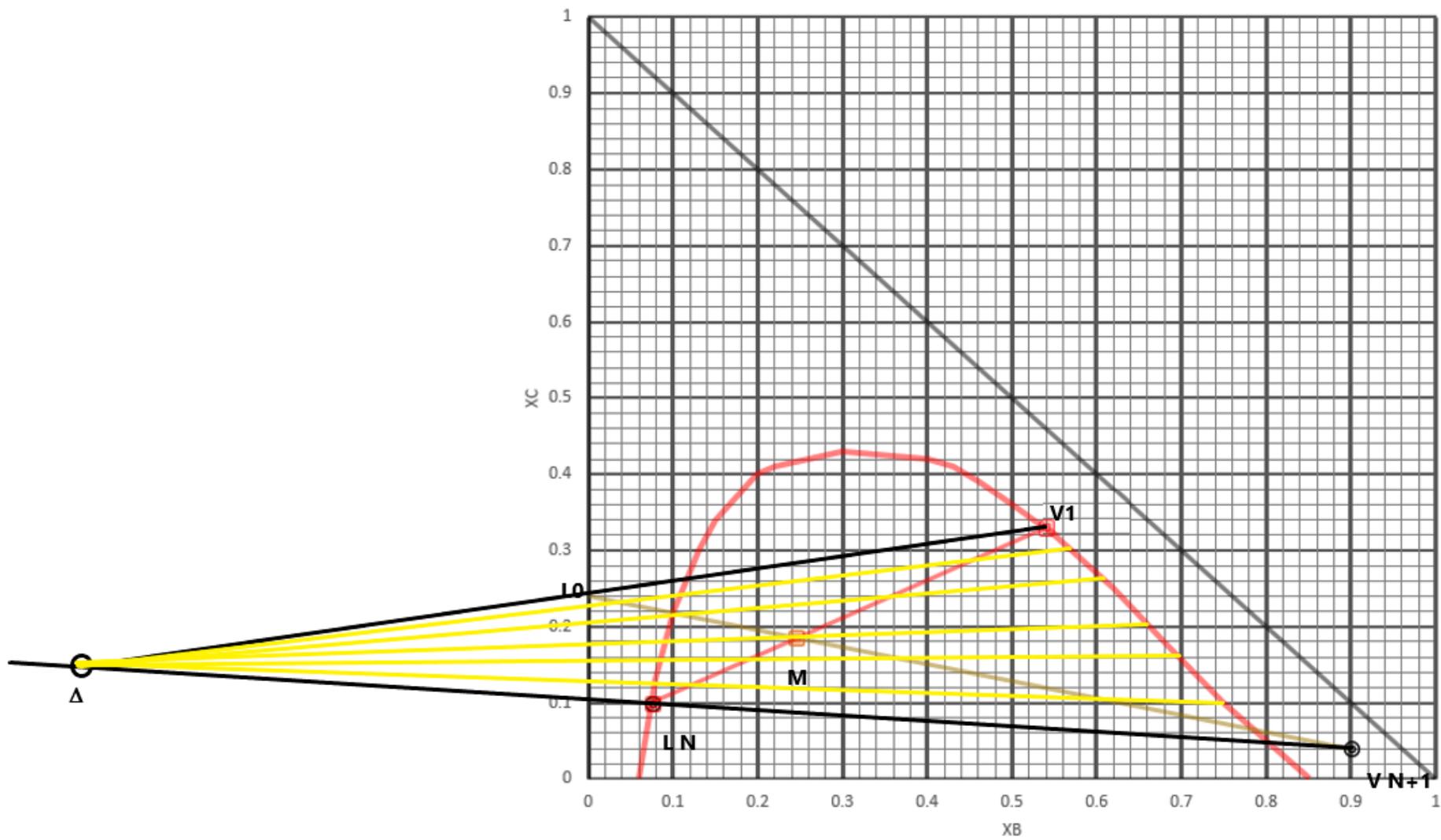
Plot this point on the line from  $L_0$  to  $V_{N+1}$  and extend line from  $LN$  through  $M$  until it contacts the solvent rich side of the phase boundary. The point  $V_1$  is at  $x_c = 0.33$ .

### Problem 01



Now draw lines  $L_N$  to  $V_{N+1}$  and  $V_1$  to  $L_0$ , extend to outside the graph to locate  $\Delta$ . Then draw a series of lines to locate points for the operative lines. (The  $x_c$  value on the raffinate side of the phase boundary is  $x$  and the  $x_c$  value on the extract side of the phase boundary is  $y$ .)

# Problem 01



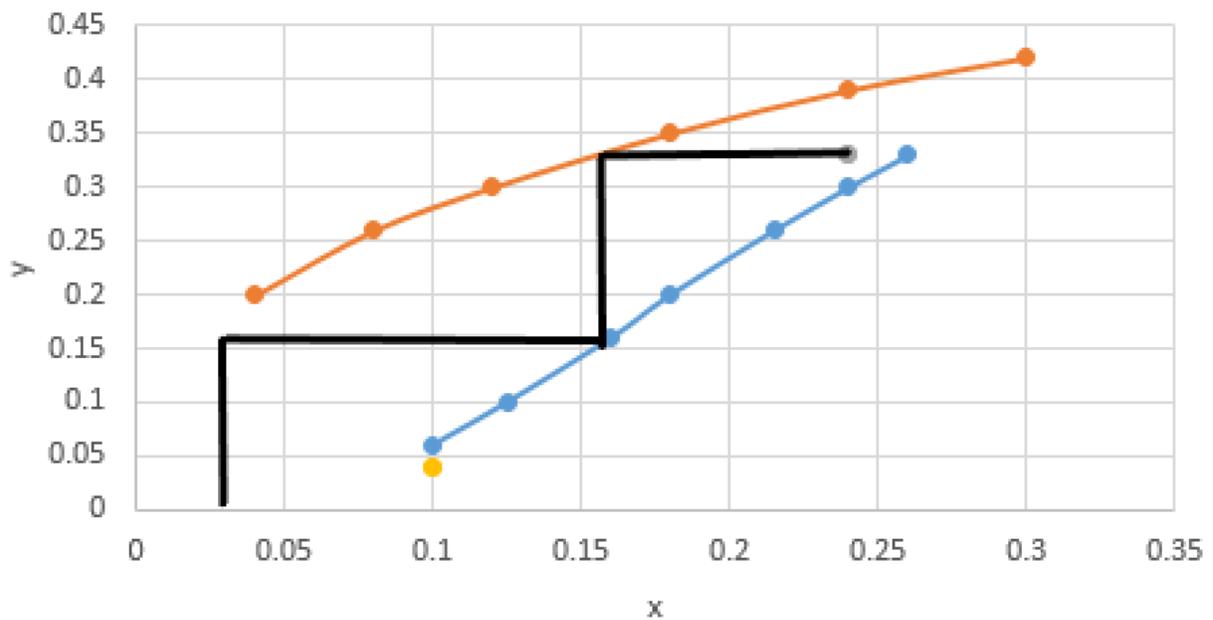
Operating Line Points	
x	y
0.26	0.33
0.24	0.30
0.215	0.26
0.18	0.20
0.16	0.16
0.125	0.10
0.10	0.06

The points for the equilibrium curve by taking  $x_c$  for the raffinate as  $x$  and  $x_c$  for the extract as  $y$  (from the equilibrium data provided) for equilibrium curve points.

Equilibrium Curve Points	
x	y
0.30	0.42
0.24	0.39
0.18	0.35
0.12	0.30
0.08	0.26
0.04	0.20

**2 stages are required.**

### McCabe-Thiele



c)

$$L_N = \frac{M}{1 + V_1/L_N}$$

$$\frac{V_1}{L_N} = \frac{x_M - x_N}{y_1 - x_M} = \frac{0.185 - 0.1}{0.33 - 0.185} = 0.586$$

$$M = F + S = 500 + 187.5 = 687.5$$

$$L_N = \frac{M}{1 + V_1/L_N} = \frac{687.5}{1 + 0.586} = 433.5 \text{ kg/hr}$$

$$V_1 = M - L_N = 687.5 - 433.5 = 254.0 \text{ kg/hr}$$

$$x_N = 0.1$$

$$y_1 = 0.33$$

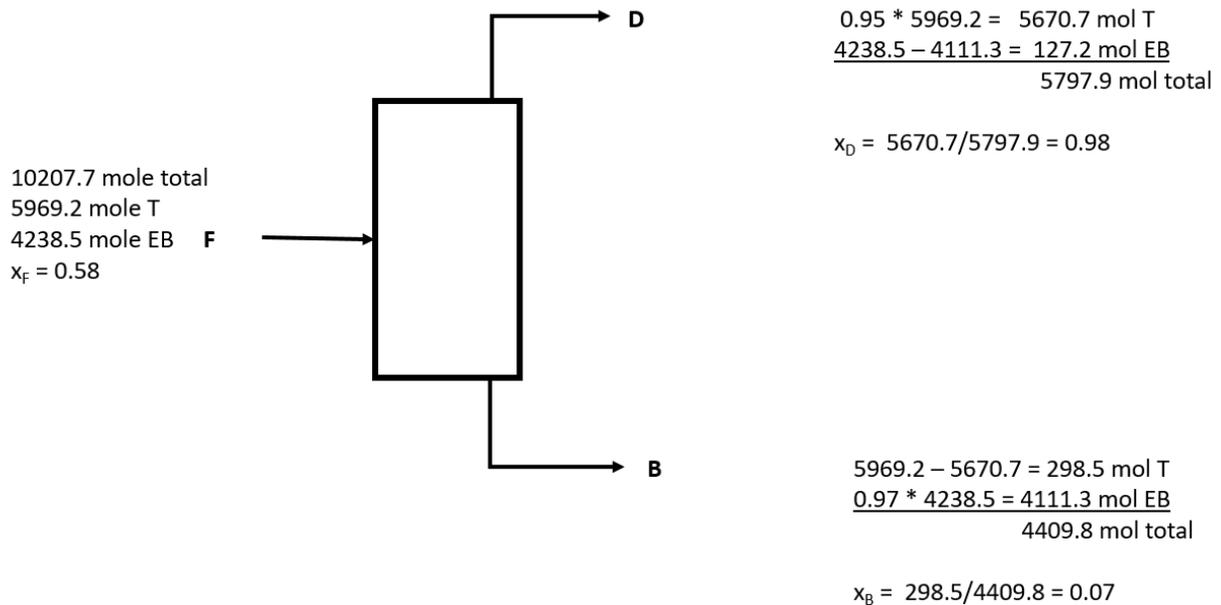
2) The first step is to convert the mass flow given into molar flows:

$$1000 \text{ kg total/hr} * 0.45 \text{ kg EB/kg total} * \frac{\text{mol EB}}{106.17 \text{ g EB}} * \frac{1000 \text{ g}}{\text{kg}} = 4238.5 \text{ mol EB/hr}$$

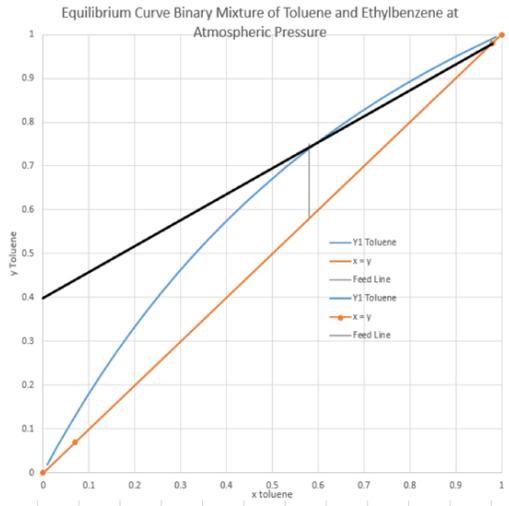
$$1000 \text{ kg total/hr} * 0.55 \text{ kg T/kg total} * \frac{\text{mol T}}{92.14 \text{ g T}} * \frac{1000 \text{ g}}{\text{kg}} = 5969.2 \text{ mol T/hr}$$

$$x_F = \frac{5969.2 \text{ mol T}}{5969.2 \text{ mol T} + 4238.5 \text{ mol EB}} = 0.58$$

1 hour basis



The next step is to determine the minimum reflux ratio:



By drawing a line from  $x_D$  to the intersection of the feed line and the equilibrium curve, the intercept can be read:

$$\frac{x_D}{R_{min} + 1} = 0.4$$

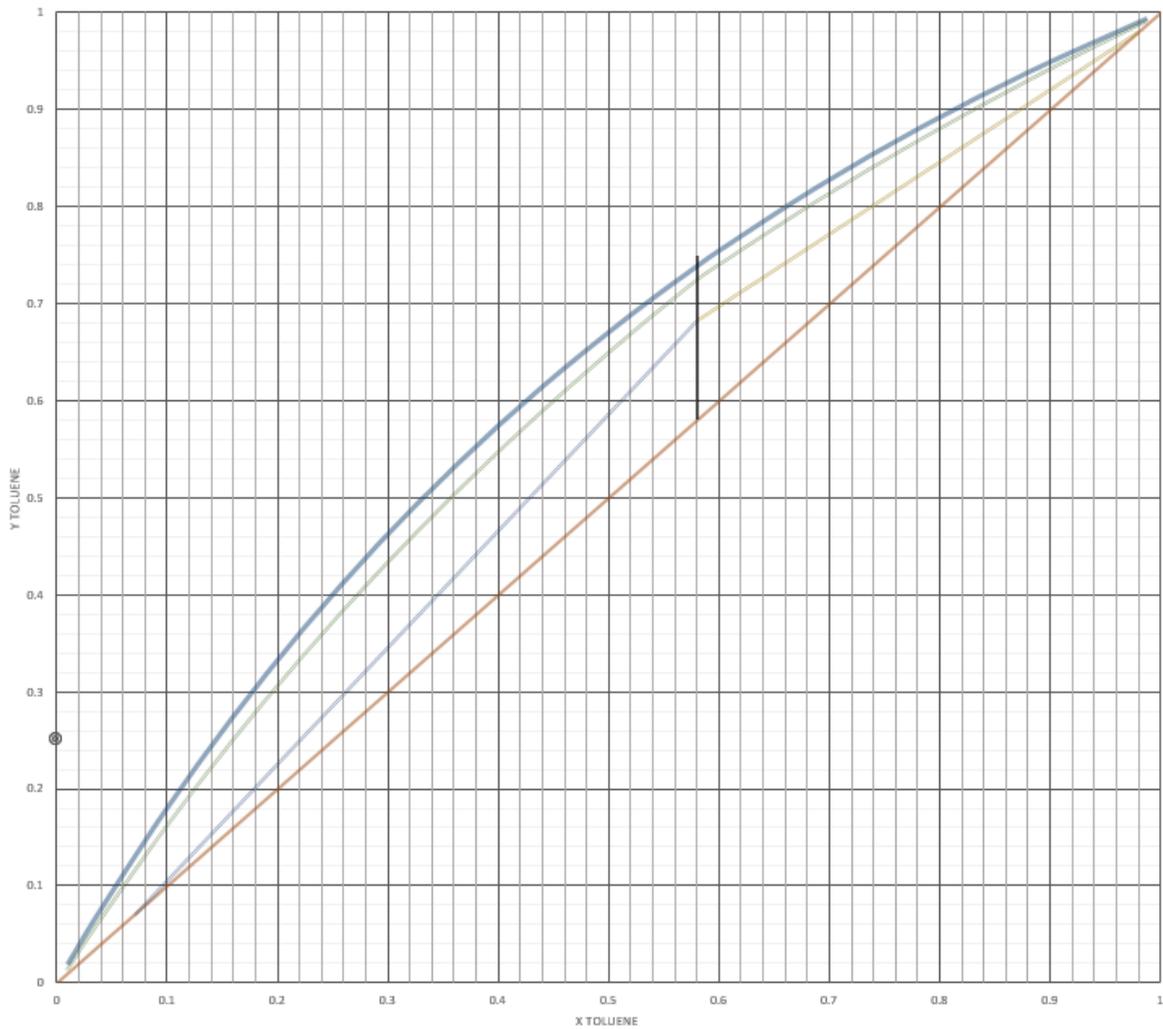
This can be solved for  $x_D = 0.98$  to give  $R_{min} = 1.45$

Problem statement says to use  $R = 2 * R_{min} = 2 * 1.45 = 2.90$

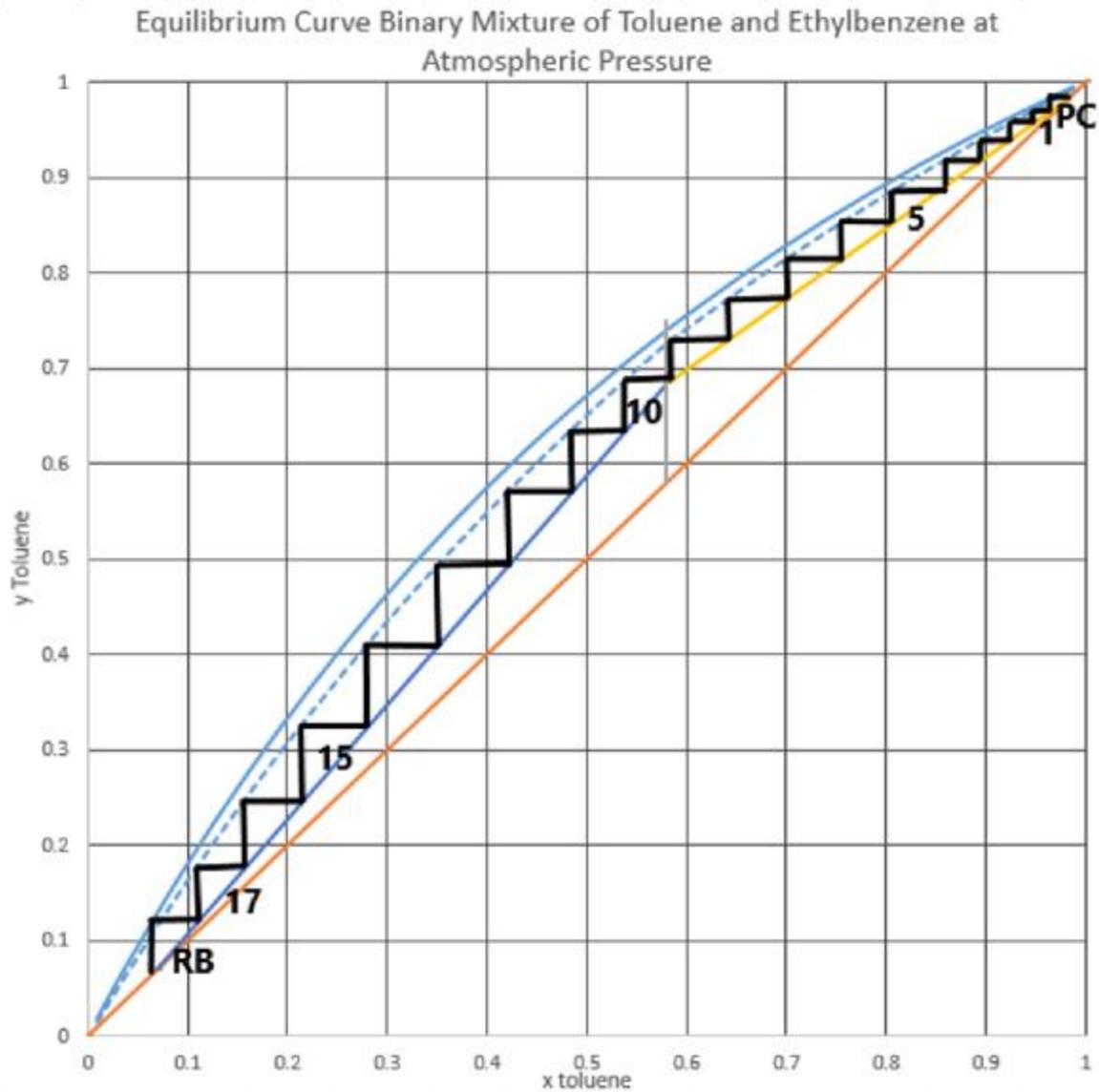
The new intercept is:

$$\frac{x_D}{R + 1} = \frac{0.98}{2.90 + 1} = 0.251$$

Equilibrium Curve Binary Mixture of Toluene and Ethylbenzene at Atmospheric Pressure



The R operating line can be drawn from the intercept to  $(x_D, x_D)$  and the S operating line can be drawn from  $(x_B, x_B)$  to the intersection of the feed line and the R operating line. The effective equilibrium curve is then estimated by marking points 75% of the way from the appropriate operating line and the equilibrium curve.



The McCabe-Thiele steps are drawn. Note that the first step is not a stage but represents the Partial Condenser.

**There are 17 stages plus the Reboiler plus the Partial Condenser**

**The optimal feed location is at the 9<sup>th</sup> or 10<sup>th</sup> stage, depending on how one drew everything...**

3) Start by converting the mass flow given to amolar flow:

$$\overline{MW} = y_{air} * MW_{air} + y_Z * MW_Z = 0.95 * 28.9 + 0.05 * 15 = 28.2 \frac{lb_m}{lb\ mol}$$

Moles entering :

$$\frac{5000\ lb_m\ total}{hr} * \frac{lb\ mol}{28.2\ lb_m} = 177.3 \frac{lb\ mol\ total}{hr}$$

Moles Air:

$$177.3 \frac{lb\ mol\ total}{hr} * 0.95 = 168.4 \frac{lb\ mol\ air}{hr}$$

Moles Zapple®:

$$177.3 \frac{lb\ mol\ total}{hr} * 0.05 = 8.9 \frac{lb\ mol\ Zapple^{\text{®}}}{hr}$$

Problem states that 90% of the Zapple® will be removed from the gas stream:

$$\text{Moles Zapple}^{\text{®}} \text{ in the gas stream at the top, } a = 0.1 * 8.9 = 0.89$$

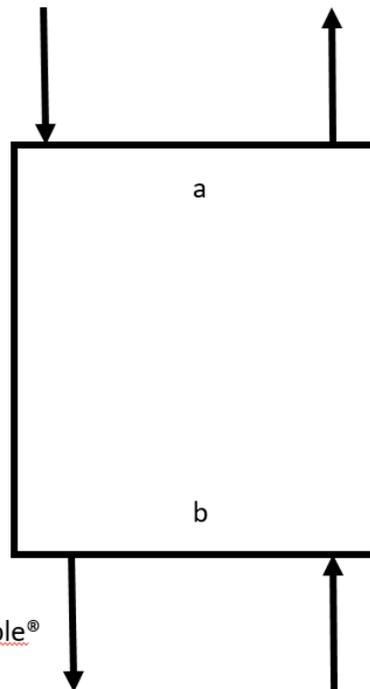
$$\text{Moles Zapple}^{\text{®}} \text{ in the liquid stream at the bottom, } b = 0.9 * 8.9 = 8.0$$

Mole fraction of Zapple® in the gas stream at the top:

$$y_a = \frac{0.89}{168.4 + 0.89} = 0.0053$$

$$L_c = ?$$

$$x_a = 0$$



$$V_c = 168.4\ lb\ mol\ air$$

$$V_{Z_a} = 0.89\ lb\ mol\ Zapple^{\text{®}}$$

$$y_a = 0.0053$$

$$L_c = \text{same}$$

$$L_{Z_b} = 8.0\ lb\ mol\ Zapple^{\text{®}}$$

$$x_b = ?$$

$$V = 177.3\ lb\ mol$$

$$V_c = 0.95 * 177.3 = 168.4\ lb\ mol\ air$$

$$V_{Z_b} = 0.05 * 177.3 = 8.9\ lb\ mol\ Zapple^{\text{®}}$$

$$y_b = 0.05$$

Because the problem statement indicates that the operating line can be considered linear we can assume that for minimum liquid flow the mole fraction of Zapple® in the exiting liquid is in equilibrium with the entering gas:

From problem statement:

$$y = 0.7 x$$

$$x_b = \frac{y_b}{0.7} = \frac{0.05}{0.7} = 0.0714$$

The mole fraction in the exiting liquid is:

$$x_b = \frac{L_{Z,b}}{L_{C,min} + L_{Z,b}} = \frac{8.0}{L_C + 8.0} = 0.0714$$

The minimum liquid flow can be solved as:

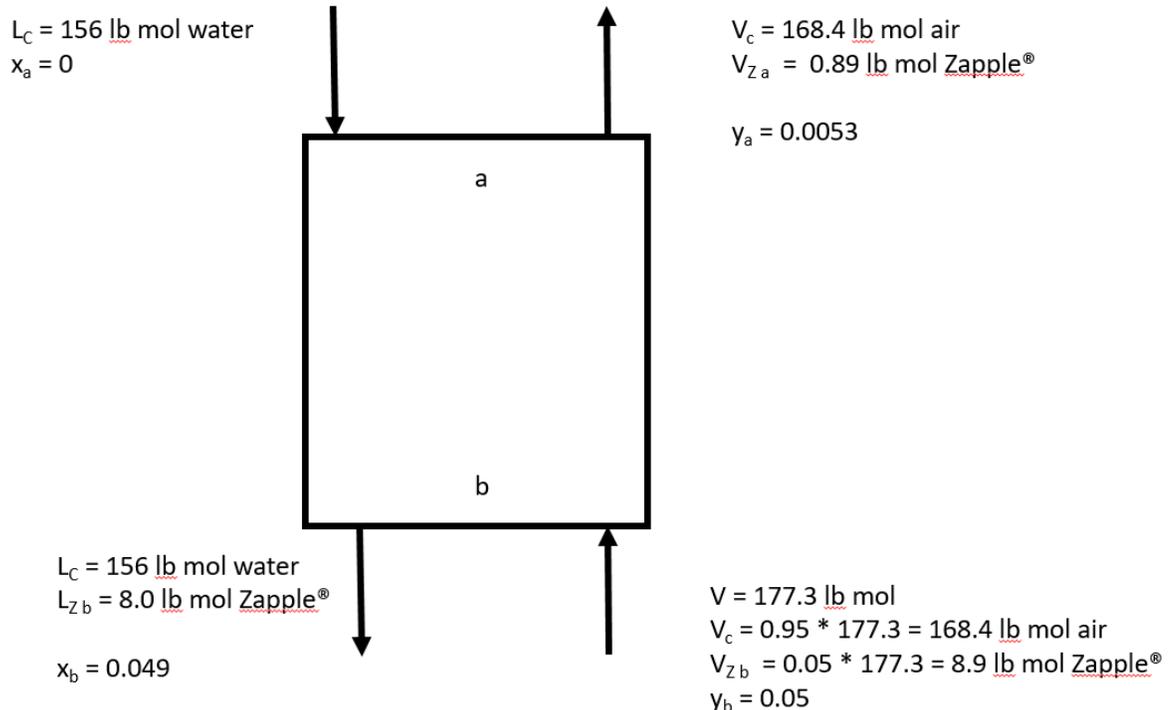
$$L_{C,min} = 104.0 \frac{\text{lb mol water}}{\text{hr}}$$

As per the problem statement, we will be using 1.5 times the minimum flow:

$$L_C = 1.5 * L_{C,min} = 1.5 * 104.0 = 156 \frac{\text{lb mol water}}{\text{hr}}$$

Now we can calculate the mole fraction in the exiting liquid:

$$x_b = \frac{L_{Z,b}}{L_C + L_{Z,b}} = \frac{8.0}{156 + 8.0} = 0.049$$



a. Now we use the flooding correlation given:

$$\Delta P_{flood} = 0.115 * F_p^{0.7}$$

From the data table given we see that for 1 1/2 " Plastic Pall Rings  $F_p = 40$  and  $f_p = 1.18$

$$\Delta P_{flood} = 0.115 * 40^{0.7} = 1.52 \frac{\text{" H2O}}{\text{ft}}$$

We are instructed to work at 50% of the flooding pressure drop, therefore we will use 0.75 " water per foot of packing.

To use the attached chart we must calculate  $\frac{G_x}{G_y} \sqrt{\frac{\rho_y}{\rho_x}}$

Although we do not know the cross-sectional area required to calculate  $G_x$  or  $G_y$ , the ratio of them is the same as the ratio of the mass flows of the liquid and vapor.

Mass flow of vapor:

$$\text{At bottom} = 5000 \text{ lb/hr}$$

$$\text{At top} = 168.4 \text{ lbmol air/hr} * 28.9 \text{ lb / lbmol} + 0.89 \text{ lbmol Z/hr} * 15 \text{ lb / lbmol} = 4880 \text{ lb/hr}$$

$$\text{Average} = 4940 \text{ lb /hr}$$

Mass flow of liquid:

$$\text{At top} = 156 \text{ lbmol water/hr} * 18 \text{ lb/lbmol} = 2808 \text{ lb/hr}$$

$$\text{At bottom} = 156 \text{ lbmol water/hr} * 18 \text{ lb/lbmol} + 8.0 \text{ lbmol/hr} * 15 \text{ lb/lbmol} = 2928 \text{ lb/hr}$$

$$\text{Average} = 2868 \text{ lb/hr}$$

$$\frac{G_x}{G_y} = \frac{2868}{4940} = 0.581$$

Note: if one had just used the values at the bottom of the tower the ratio would be 0.562, very similar...

The density of the liquid can be approximated as the density of water,

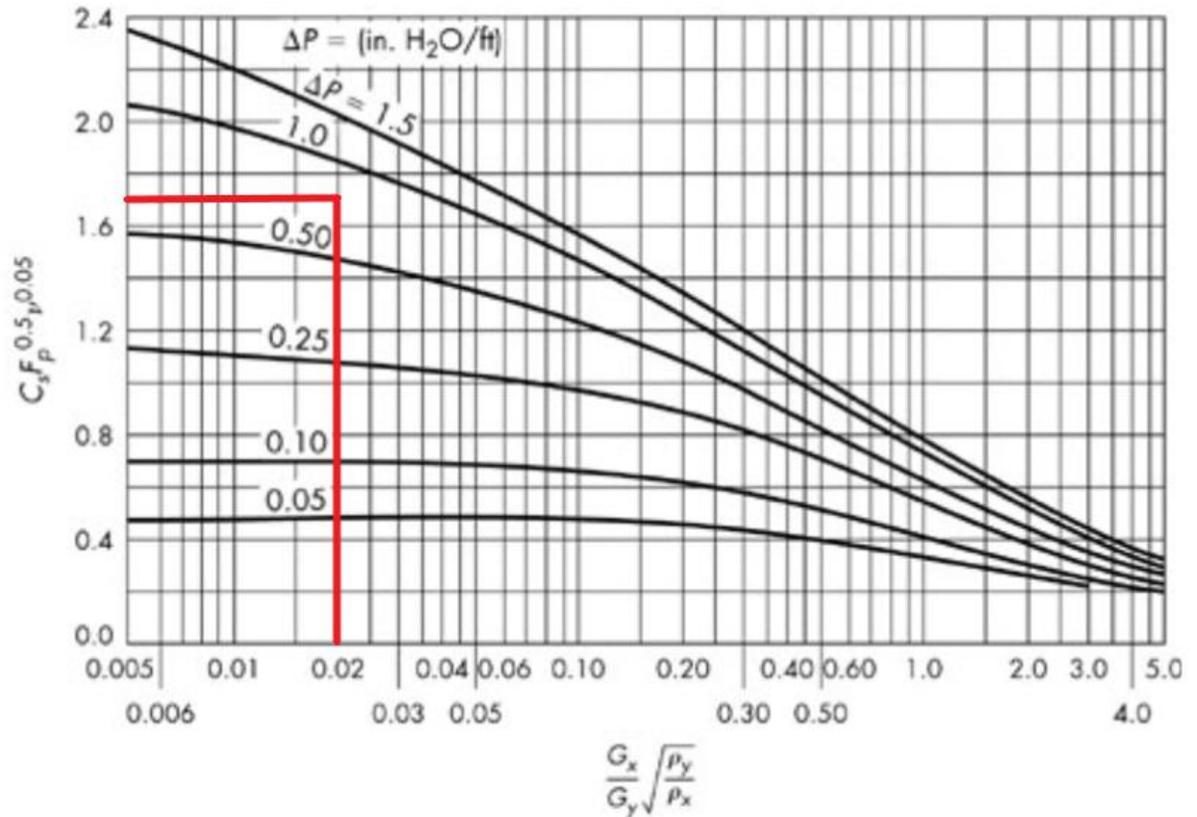
$$\rho_x = 62.4 \text{ lb/ft}^3$$

The density of the vapor can be obtained using the ideal gas law:

$$\rho_y = \frac{\overline{MWP}}{RT} = \frac{28.2 \frac{\text{lb}}{\text{lbmol}} 1 \text{ atm}}{0.73024 \frac{\text{ft}^3 \text{ atm}}{\text{R lbmol}} 527.67 \text{ R}} = 0.07318 \frac{\text{lb}}{\text{ft}^3}$$

Now:

$$\frac{G_x}{G_y} \sqrt{\frac{\rho_y}{\rho_x}} = 0.581 \sqrt{\frac{0.07318}{62.4}} = 0.0199$$



From graph we can read (by interpolating between the lines from 0.5 and 1.0 “water/ft) that

$$C_s F_p^{0.5} v^{0.05} = 1.7$$

$$C_s 40_p^{0.5} 1^{0.05} = 17$$

$$C_s = 0.269$$

By definition:

$$C_s = u_0 \sqrt{\frac{\rho_y}{\rho_x - \rho_y}}$$

$$0.269 = u_0 \sqrt{\frac{0.07318}{62.4 - 0.07318}} = 0.0323 u_0$$

$$u_0 = 7.85 \text{ ft/s}$$

Calculate largest volumetric flow:

$$5000 \frac{lb}{hr} * \frac{ft^3}{0.07318 lb} * \frac{hr}{3600 s} = 19.0 \frac{ft^3}{s}$$

The required area is:

$$Area = \frac{volumetric\ flow}{linear\ velocity} = \frac{19.0 \frac{ft^3}{s}}{7.85 ft/s} = 2.42 ft^2 = \frac{\pi D^2}{4}$$

$$D = 1.75 ft$$

b) Now that we have the cross-sectional area we can calculate  $G_x$  and  $G_y$  independently:

$$G_x = \frac{2868 \frac{lb}{hr}}{2.42 ft^2} = 1185 \frac{lb}{ft^2 hr}$$

$$G_y = \frac{4940 \frac{lb}{hr}}{2.42 ft^2} = 2041 \frac{lb}{ft^2 hr}$$

We can now calculate the height of the transfer unit:

$$H_x = 0.9 ft \left( \frac{G_x/\mu}{1500 \frac{lb}{ft^2 hr} / 0.891 cP} \right)^{0.3} \left( \frac{S_c}{381} \right)^{0.5} \frac{1}{f_p}$$

$$H_x = 0.9 ft \left( \frac{1185 \frac{lb}{ft^2 hr} / 0.891 cP}{1500 \frac{lb}{ft^2 hr} / 0.891 cP} \right)^{0.3} \left( \frac{350}{381} \right)^{0.5} \frac{1}{1.18} = 0.68 ft$$

$$H_y = 1.4 ft \left( \frac{G_y}{500 \frac{lb}{ft^2 hr}} \right)^{0.3} \left( \frac{1500 \frac{lb}{ft^2 hr}}{G_x} \right)^{0.4} \left( \frac{S_c}{0.66} \right)^{0.5} \frac{1}{f_p}$$

$$H_y = 1.4 ft \left( \frac{2041}{500 \frac{lb}{ft^2 hr}} \right)^{0.3} \left( \frac{1500 \frac{lb}{ft^2 hr}}{1185} \right)^{0.4} \left( \frac{0.75}{0.66} \right)^{0.5} \frac{1}{1.18} = 2.12 ft$$

Now we can calculate height of overall transfer unit:

$$H_{Oy} = H_y + m \frac{V}{L} * H_x$$

From problem statement we know that  $y = 0.7 x$  and therefore  $m = 0.7$

For  $V/L$  we can use:

$$V/L = \frac{x_b - x_a}{y_b - y_a} = \frac{0.049 - 0}{0.05 - 0.0053} = 1.10$$

Had one used the molar flows at the bottom of the tower  $V/L = 1.08$  would be obtained.

Now:

$$H_{Oy} = 2.12 \text{ ft} + 0.7 * 1.10 * 0.68 \text{ ft} = 2.64 \text{ ft}$$

Now we will calculate the number of transfer units:

$$N_{Oy} = \frac{y_b - y_a}{(y - y^*)_{lm}}$$

$$y_a = 0.0053$$

$$y_b = 0.05$$

$$y_a^* = 0.7 * x_a = 0.7 * 0 = 0$$

$$y_b^* = 0.7 * x_b = 0.7 * 0.049 = 0.0343$$

$$y_b - y_a = 0.05 - 0.0053 = 0.0447$$

$$y_a - y_a^* = 0.0053 - 0 = 0.0053$$

$$y_b - y_b^* = 0.05 - 0.0343 = 0.0157$$

$$\frac{1}{(y - y^*)_{lm}} = \frac{(y_a - y_a^*) - (y_b - y_b^*)}{\ln \frac{y_a - y_a^*}{y_b - y_b^*}} = \frac{0.0053 - 0.0157}{\ln \frac{0.0053}{0.0157}} = \frac{-0.0104}{-1.0860} = 0.00958$$

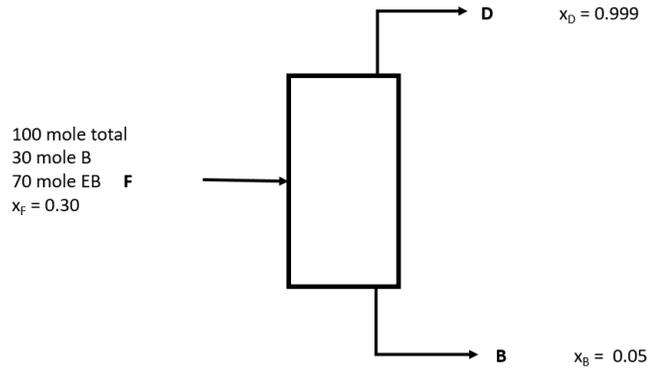
$$N_{Oy} = \frac{y_b - y_a}{(y - y^*)_{lm}} = \frac{0.0447}{0.00958} = 4.67$$

The required height of the packing can be calculated as:

$$Z_t = H_{Oy} * N_{Oy} = 2.64 \text{ ft} * 4.67 = 12.3 \text{ ft}$$

4)

1 minute basis



Reflux ratio is given as 1.5

$$\text{a) } D = F \left( \frac{x_F - x_B}{x_D - x_B} \right) = 100 * \left( \frac{0.3 - 0.05}{0.999 - 0.05} \right) = 26.34 \text{ mol/min}$$

$$B = F \left( \frac{x_D - x_F}{x_D - x_B} \right) = 100 * \left( \frac{0.999 - 0.3}{0.999 - 0.05} \right) = 73.66 \text{ mol/min}$$

b) and c)

The R Operating line has the following equation:

$$y_{n+1} = \frac{R}{R+1} x_n + \frac{x_D}{R+1}$$

$$y_{n+1} = \frac{1.5}{1.5+1} x_n + \frac{0.999}{1.5+1}$$

$$y_{n+1} = 0.6000000 x_n + 0.3996000$$

Use the Kremser Equation to determine the number of stages required to go from  $x = 0.9$  to  $x = 0.999$

We need to determine the equation for the line which approximates the equilibrium curve in this region:

Line passes through the points (1,1) and (0.9000000, 0.9815065)

The slope will be (rise over run):

$$m = \frac{1 - 0.9815065}{1 - 0.9000000} = 0.1849350$$

The intercept can be evaluated for the point (1,1)

$$y = 0.1849350 x + b$$

$$1 = 0.1849350 * (1) + b$$

$$b = 0.8150650$$

$$y = 0.1849350 x + 0.8150650 \quad \text{Equilibrium Line}$$

Data needed for Kremser Equation:

a evaluated at  $x = x_D$

b evaluated at  $x = 0.9$

At  $x = x_D = 0.999$  the value on the operating line is equal to  $x_D = 0.999$

$$y_a = 0.6000000 * 0.999 + 0.399600 = 0.9990000$$

At  $x = 0.90$  the value on the operating line is

$$y_b = 0.6000000 * 0.9 + 0.399600 = 0.9396000$$

At  $x = x_D = 0.999$  the value on the equilibrium line is

$$y_a^* = 0.1849350 * 0.999 + 0.8150650 = 0.9998151$$

At  $x = 0.90$  the value on the equilibrium line is (this value is also known from the problem statement)

$$y_b^* = 0.1849350 * 0.9 + 0.8150650 = 0.9815065$$

The number of stages required to get from  $x = 0.9$  to  $x = 0.999$  is:

$$N = \frac{\ln[(y_b - y_b^*)/(y_a - y_a^*)]}{\ln[(y_b - y_a)/(y_b^* - y_a^*)]}$$

$$y_b - y_b^* = -0.0419065$$

$$y_a - y_a^* = -0.0008151$$

$$y_b - y_a = -0.0594000$$

$$y_b^* - y_a^* = -0.0183086$$

$$(y_b - y_b^*)/(y_a - y_a^*) = -0.0419065 / -0.0008151 = 51.41271$$

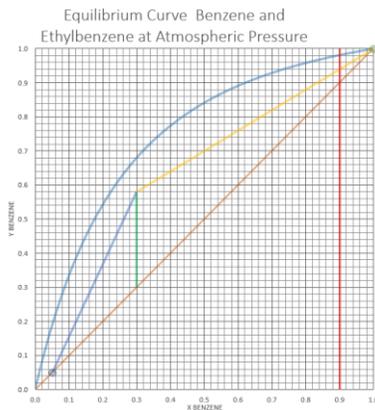
$$(y_b - y_a)/(y_b^* - y_a^*) = -0.0594000 / -0.0183086 = 3.244377$$

$$N = \frac{\ln[(y_b - y_b^*) / (y_a - y_a^*)]}{\ln [(y_b - y_a) / (y_b^* - y_a^*)]} = \frac{\ln[51.41271]}{\ln [3.244377]} = \frac{3.939885}{1.176923} = 3.35 \text{ stages}$$

Round up to 4 stages

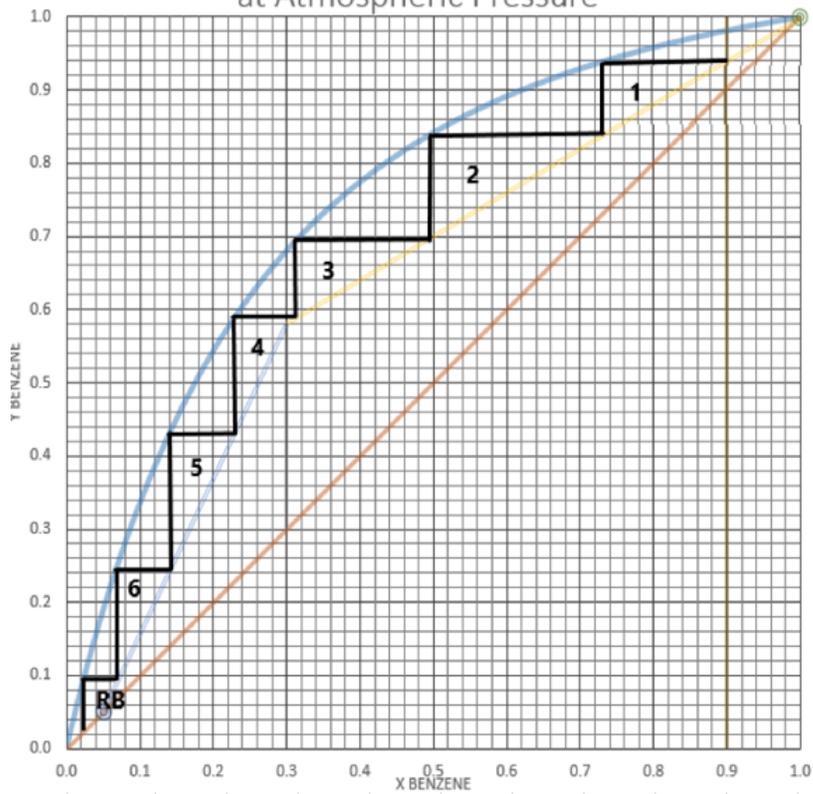
Now on McCabe-Thiele diagram draw R operating line from  $(x_D, x_D)$  to the intercept calculated earlier, 0.4 (we don't need all of the significant digits used earlier).

Then draw the S operating line from  $(x_B, x_B)$  to the intersection of the equilibrium curve and the feed line. Because the feed entered as a saturated liquid the feed line is a vertical line at  $x = 0$ .



The steps can then be drawn in:

### Equilibrium Curve Benzene and Ethylbenzene at Atmospheric Pressure



The required number of steps is  $4 + 6 + \text{Reboiler}$ .

Therefore 10 stages plus reboiler.

The feed enters on stage  $4 + 4 = \text{Stage } 8$